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A comparative study of the LBE and GKS methods for 2D near incompressible laminar flows

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Abstract

We compare the lattice Boltzmann equation (LBE) and the gas-kinetic scheme (GKS) applied to 2D incompressible laminar flows. Although both methods are derived from the Boltzmann equation thus share a common kinetic origin, numerically they are rather different. The LBE is a finite difference method, while the GKS is a finite-volume one. In addition, the LBE is valid for near incompressible flows with low-Mach number restriction $Ma < 0.3$, while the GKS is valid for fully compressible flows. In this study, we use the generalized lattice Boltzmann equation (GLBE) with multiple-relaxation-time (MRT) collision model, which overcomes all the apparent defects in the popular lattice BGK equation. We use both the LBE and GKS methods to simulate the flow past a square block symmetrically placed in a 2D channel with the Reynolds number Re between 10 and 300. The LBE and GKS results are validated against the well-resolved results obtained using finite-volume method. Our results show that both the LBE and GKS yield quantitatively similar results for laminar flow simulations, and agree well with existing ones, provided that sufficient grid resolution is given. For 2D problems, the LBE is about 10 and 3 times faster than the GKS for steady and unsteady flow calculations, respectively, while the GKS uses less memory. We also observe that the GKS method is much more robust and stable for under-resolved cases due to its upwinding nature and interpolations used in calculating fluxes.

Keywords: Lattice Boltzmann equation; Gas-kinetic scheme; Linearized Boltzmann equation; Incompressible flows; 2D flow past a square in a channel

1. Introduction

In general, kinetic methods for computational fluid dynamics (CFD) are derived from the Boltzmann equation, as opposed to conventional CFD methods based on direct discretizations of the Navier–Stokes equations. Two distinctive features of kinetic methods immediately appear. First, kinetic methods can include

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extended hydrodynamics beyond the validity regime of the Navier–Stokes equations, because they are based on kinetic theory. It is known that the Boltzmann equation provides the theoretical connection between hydrodynamics and the underlying microscopic physics. Kinetic methods are often called mesoscopic methods for they act between the macroscopic conservation laws and their corresponding microscopic dynamics. And second, the Boltzmann equation is a first-order integro-partial-differential equation with a linear advection term, while the Navier–Stokes equation is a second-order partial-differential equation with a nonlinear advection term. The nonlinearity in the Boltzmann equation resides in its collision term, which is local. This feature may lead to some computational advantages [1]. For these reasons, kinetic methods have attracted some interest recently. Due to their mesoscopic nature, kinetic methods are particularly appealing in modeling and simulations of complex fluids (cf. [2] and references therein).

There are a number of kinetic or mesoscopic methods, such as the lattice gas cellular automata (LGCA), the lattice Boltzmann equation (LBE), the gas-kinetic schemes (GKS), the smoothed particle hydrodynamics (SPH), and the dissipative particle dynamics (DPD). Among these methods, the LBE and GKS methods are specifically designed as numerical methods for CFD: the former is only valid for near incompressible flows while the latter is for fully compressible flows. Both methods have been applied to simulate viscous flows [3,4], heat transfer problems [5,6], shallow water equations [7], multiphase [8–11] and multi-component [12–17] flows, magnetohydrodynamics [18–20], and microflows [21–24]. Besides their common connections to the Boltzmann equation, the LBE and GKS methods are quite different in many ways. First of all, the LBE is a finite difference (FD) method while the GKS is a finite-volume (FV) one. Second, the LBE is a system evolving on the discrete phase space $\Gamma^* := (\delta x\mathbb{Z}^d, \{\boldsymbol{c}_i\})$ and in discrete time $t_n \in \delta t\mathbb{N}_0 := \delta t\{0, 1, 2, \ldots\}$, where $\delta x\mathbb{Z}^d$ is a d-dimension lattice space with a lattice spacing δx, $\{\boldsymbol{c}_i | i = 0, 1, 2, \ldots, N\}$ is a finite discrete velocity set, and δt is the time step size. In the LBE, the phase space $\Gamma := (\boldsymbol{x}, \boldsymbol{\xi})$ and time t are discretized in such way that for any lattice point $\boldsymbol{x}_j \in \delta x\mathbb{Z}^d$, $\boldsymbol{x}_j + \boldsymbol{c}_i\delta t \in \delta x\mathbb{Z}^d$ for all $\boldsymbol{c}_i \in \{\boldsymbol{c}_i\}$. In contrast, the particle velocity space $\boldsymbol{\xi}$ remains continuous in the GKS, and the space $\boldsymbol{x}$ and time t are discretized independently. However, the grid spacing Δx and time step size Δt in the GKS must satisfy certain stability criteria. Finally and most importantly, in the LBE, the Maxwellian equilibrium distribution function is approximated by its low-order Taylor expansion about zero flow velocity. This approximation limits the LBE method to near incompressible flows with low-Mach number restriction $Ma < 0.3$. The GKS method uses the Maxwellian equilibrium distribution function without any approximation, and it is valid for fully compressible flows.

A comparison between the GKS and LBE methods has been made previously [25]. However, the previous study [25] has a very limited scope. First, the GKS was compared with the lattice Bhatnagar–Gross–Krook [26] (BGK) equation or LBGK model [27,28], which has some severe defects leading to numerical instability and inaccurate boundary conditions [29,30]. Second, the previous study is limited to steady internal flow (the 2D lid-driven cavity flow). And third, only the numerical accuracy of both methods are compared in the previous study, but the numerical stability and computational efficiency of these two methods were not investigated.

The present work is motivated to provide a more thorough comparative study of the LBE and GKS methods for simulations of near incompressible flows. The present work differs from the previous study in several aspects. First of all, instead of the popular LBGK equation [27,28], we shall use the generalized lattice Boltzmann equation (GLBE) with multiple-relaxation-time (MRT) collision model [31]. The GLBE or MRT-LBE can overcome *all* the apparent defects in the LBGK equation. Second, test case used in this work is an open flow involving boundary conditions different from the ones in internal flows. Finally, we will investigate in detail the numerical accuracy, stability and efficiency of both the LBE and GKS methods for low-Mach number laminar flows. We will also validate both the LBE and GKS methods by comparing our results with the existing ones obtained by other well-established methods.

The remaining part of this paper is organized as follows. We provide first a succinct discussion on kinetic theory, especially on the linearized Boltzmann equation, and then the formulations of the GKS and LBE methods in Section 2. We discuss the test flow problem in detail in Section 3. The test case we choose is the 2D flow past a square block symmetrically placed in a channel [32]. We discuss two methods to evaluate hydrodynamic forces: the pressure-tensor integration method and the momentum-exchange method, used in the GKS and LBE simulations, respectively. We report the numerical results with the Reynolds number Re between 10 and 300 in Section 4, which include both steady and unsteady flows. We compare velocity fields,