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Shallow water model on cubed-sphere by multi-moment finite volume method

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Abstract

A global numerical model for shallow water flows on the cubed-sphere grid is proposed in this paper. The model is constructed by using the constrained interpolation profile/multi-moment finite volume method (CIP/MM FVM). Two kinds of moments, i.e. the point value (PV) and the volume-integrated average (VIA) are defined and independently updated in the present model by different numerical formulations. The Lax–Friedrichs upwind splitting is used to update the PV moment in terms of a derivative Riemann problem, and a finite volume formulation derived by integrating the governing equations over each mesh element is used to predict the VIA moment. The cubed-sphere grid is applied to get around the polar singularity and to obtain uniform grid spacing for a spherical geometry. Highly localized reconstruction in CIP/MM FVM is well suited for the cubed-sphere grid, especially in dealing with the discontinuity in the coordinates between different patches. The mass conservation is completely achieved over the whole globe. The numerical model has been verified by Williamson's standard test set for shallow water equation model on sphere. The results reveal that the present model is competitive to most existing ones.

Keywords: Finite volume method; Cubed-sphere grid; Shallow water equations; Spherical geometry; Global model; High-order scheme; Geophysical flows

1. Introduction

How to represent the spherical geometry is one of the key points to the numerical simulations on the large scale circulation of the Earth's atmosphere. In the context of finite difference formulation, latitude–longitude (LAT/LON) grid has gained a greater popularity compared to other grids for spherical geometry, such as the icosahedron geodesic grid [30,35] and the cubed-sphere grid [29] which were proposed more than 30 years ago though. The standard LAT/LON grid, however, meets substantial difficulties in regions close

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to the poles, i.e. the singularities in the governing equations and the convergence of meridians. Although the numerical barriers in the LAT/LON grid can be circumvented to some extent by using special numerical techniques, for example the polar cap [9], the semi-Lagrangian/semi-implicit formulation [5] and the reduced LAT/LON grid [26], the large ratio in the grid spacing seriously prevents the uniformly resolved numerical solutions on the whole sphere. Moreover, these problems are getting worse as the spatial resolution is increasingly refined.

Other grids with more uniform resolution for the whole globe began to draw more attention since the middle of 1990's, an age from which the GCM simulations with high resolution became technically possible due to the rapid development of computer hardware. Two representatives of the grids for spherical geometry, i.e. the gnomonic or conformal cubic grid [18,19,25,27,28] and the icosahedron geodesic grid [8,16,31,34] have been adopted in global models for either shallow water models or atmospheric models. The cubed-sphere grid is generated by mapping the sphere onto an inscribed cube using the gnomonic projection. A expanded spherical cube is composed of six identical patches connected to each other by 12 edges which are also called patch boundaries. A gnomonic projection results in a more uniform grid spacing, but the mesh on each patch is not orthogonal. Furthermore, the coordinate across a patch boundary is not continuous. As commentated in [25], the "breaking" of the coordinate along the patch boundaries requires extra numerical treatments that usually degrade the accuracy of numerical solution. A remedy to the non-orthogonality of the gnomonic cubic grid is the conformal cubic grid [25]. McGregor [18] implemented the semi-Lagrangian method on the conformal cubic grid and constructed a global atmospheric model with some special numerical technique [20].

More accurate numerical formulations on the cubed-sphere grid have been recently devised by implementing some advanced numerical methods, i.e. the spectral element method [7,32] and the discontinuous Galerkin (DG) method [23,24]. The DG method [4] computes the volume-integrated value over each control volume (mesh cell) via a finite volume formulation, which then guarantees the numerical conservation. High order reconstruction can be built locally in a DG method by increasing the local degrees of freedom. The DG method, however, involves numerical quadrature which is computationally expensive, and moreover, the CFL condition for a high order DG method is much restrictive as discussed in [44].

We have recently developed another type of high resolution scheme, namely constrained interpolation profile/multi-moment finite volume method (CIP/MM FVM) for fluid dynamic simulations. Different from the conventional finite volume methods, the CIP/MM FVM employs at least two kinds of quantities which are generically called "moment" in our context, e.g. the point value (PV) and the volume-integrated averages (VIA) of a field variable, as the prognostic variables. The moments are put forward in time separately using different numerical algorithms. For example, the PV is updated by solving a point-wise Riemann problem or a semi-Lagrangian procedure, while the VIA has to be computed via a finite volume scheme of flux-form to assure the conservation. A CIP/MM FVM allows larger CFL number for stability and is more computationally efficient. The interpolation reconstruction, which is required for the computations of the Riemann problem and the numerical flux, is built in terms of both PV and VIA, thus the mesh stencil used in the reconstruction is very compact. Using multi-moments as the model variables in constructing FVM has been implemented to CFD problems so far in [10,11,37,38]. The multi-moment formulation may result in numerical dispersion property different from the conventional finite difference methods. In [39], for example, the numerical dispersion of the simplest multi-moment finite volume method for the geostrophic adjustment was discussed. In [10], a high order advection scheme was developed for unstructured grid where a cubic interpolation function is constructed over a single triangular mesh by using both PV and VIA moments. The local reconstruction makes the CIP/MM FVM well suited not only for the unstructured grid but also for the numerical treatment of the patch boundary in the cubed-sphere grid.

We present in this paper a CIP/multi-moment finite volume scheme for shallow water equation model on cubed-sphere grid with equiangular gnomonic projection. The PV moment is defined at the vertices and the middle points of the boundary edges, while the VIA is defined over the 2D quadrilateral control volume for each mesh element. Only one layer of ghost cells are required across the patch boundaries during updating PV. The computation over the patch boundaries of the gnomonic cube can be carried out by mapping the ghost cells from the neighboring patch.