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A hybrid-Vlasov model based on the current advance method for the simulation of collisionless magnetized plasma

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Abstract

We present a numerical scheme for the integration of the Vlasov–Maxwell system of equations for a non-relativistic plasma, in the hybrid approximation, where the Vlasov equation is solved for the ion distribution function and the electrons are treated as a fluid. In the Ohm equation for the electric field, effects of electron inertia have been retained, in order to include the small scale dynamics up to characteristic lengths of the order of the electron skin depth. The low frequency approximation is used by neglecting the time derivative of the electric field, i.e. the displacement current in the Ampere equation.

The numerical algorithm consists in coupling the splitting method proposed by Cheng and Knorr in 1976 [C.Z. Cheng, G. Knorr, J. Comput. Phys. 22 (1976) 330–351.] and the current advance method (CAM) introduced by Matthews in 1994 [A.P. Matthews, J. Comput. Phys. 112 (1994) 102–116.] In its present version, the code solves the Vlasov–Maxwell equations in a five-dimensional phase space (2-D in the physical space and 3-D in the velocity space) and it is implemented in a parallel version to exploit the computational power of the modern massively parallel supercomputers. The structure of the algorithm and the coupling between the splitting method and the CAM method (extended to the hybrid case) is discussed in detail. Furthermore, in order to test the hybrid-Vlasov code, the numerical results on propagation and damping of linear ion-acoustic modes and time evolution of linear elliptically polarized Alfvén waves (including the so-called whistler regime) are compared to the analytical solutions. Finally, the numerical results of the hybrid-Vlasov code on the parametric instability of Alfvén waves are compared with those obtained using a two-fluid approach.

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1. Introduction

The understanding of the complex phenomena observed in natural and laboratory plasmas requires the use of numerical simulations. In many cases the collisional mean free path of particles is much longer than the typical scale lengths involved in these phenomena. Therefore, the plasma can be considered as collisionless and the Vlasov equation is used to investigate the behavior of the system. The most widely used method to describe numerically the kinetic dynamics of a plasma system is the particle in cell (PIC) method, essentially a Monte-Carlo method [3]. Nevertheless, Eulerian Vlasov codes, based on the direct numerical integration of the particle distribution function, have nowadays become largely adopted.

PIC codes represent historically the most adopted approach to numerical simulations of plasmas in the framework of the kinetic theory. PIC simulations follow the particle dynamics through a Lagrangian approach, i.e. integrating numerically the equation of motion for a large number of macro-particles, under the effect of the external electromagnetic fields and/or the self-consistent fields, i.e. the ones created by the particle dynamics itself. The distribution of particles is described in a statistical way where "real particles" are represented by so called super-particles. Each super-particle represents a given percentage of density and carries given amount of momentum. The density and momentum are distributed among macro-particles in such a way that their weighted summation at a given point of the spatial domain provides a mean value of the given moment of the particle distribution function with certain accuracy determined by the statistical properties of the set of macro-particles.

On the other hand, the Vlasov approach [4] is an Eulerian approach, where Vlasov equation is numerically solved by calculating the value of the particle distribution function at each time step on an uniform fixed grid used to sample the phase space. Moments needed for the time integration of the Maxwell equations are evaluated by straight numerical integration of the distribution function. Such Eulerian approach avoids the statistical noise caused by the fact that macro-particles in PIC codes represent values of the distribution function at randomly selected points of the phase space. It is necessary, however, to point out that the Eulerian Vlasov algorithm presents some limitations due to the large amount of computing resources and execution time needed to advance the distribution function.

To reduce these computational limitations, one can adopt several approximations to simplify the physical scenario, as the high frequency approximation that neglects the ion dynamics, or the hybrid approximation [2] where, for relatively large spatial scales and low frequencies, the detailed behavior of the electrons is irrelevant and the ions dominate the dynamics of the system. In the hybrid approximation, the ions are treated as kinetic particles whereas the electrons are represented as massless fluid. The hybrid scheme [2] was introduced to simplify the description of the electron dynamics, in order to eliminate their fast and small scale dynamics. As a consequence, larger time steps can be used, like fractions of the local inverse of the ion gyrofrequency Ω_{ci}^{-1} (typical for ion dynamics). However, in several problems (see, for example, study of reformation of one-dimensional perpendicular shocks [5]) the standard hybrid approximation is too restrictive and may lead to unphysical conclusions, since hybrid codes use a MHD framework where electrons are reduced to a massless, isothermal, isotropic fluid.

On the other hand, the electron inertial scales, not included by these numerical models, can be of crucial importance in many processes in different physical environments. For example, it is shown in [5] that the maximum gradient $\mathrm{d}B_y/\mathrm{d}x/B_0$ in the shock front simulated by the hybrid code without the capability of resolving the electron inertial scales strongly grows with decreasing spatial resolution $\mathrm{d}x$, while it is expected such growth to stop at the electron inertial scales. It is also known that the typical scale of the shock thickness and the typical scale of the upstream dispersive whistler wave trains is the electron inertial length c/ω_{pe} (where c is speed of light and ω_{pe} is electron plasma frequency). The electron inertial scales can be naturally introduced in a hybrid code by considering the Ohm's law in a more general form by keeping the electron mass as finite. For an introduction to the topic of a hybrid model, see [6,7] and for a more detailed description of the hybrid models with different approximation of the electron fluid, see [8].

In this paper we present a numerical code which combines a kinetic Vlasov description of ions with a fluid description for electrons, going from the MHD to the EMHD time and length scales. In particular, we use the current advance method with cyclic leapfrog (CAM-CL) for the calculation of electromagnetic fields intro-