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A transient higher order compact scheme for incompressible viscous flows on geometries beyond rectangular

Swapan K. Pandit *, Jiten C. Kalita, D.C. Dalal

Department of Mathematics, Indian Institute of Technology Guwahati, Guwahati 781 039, India

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Abstract

In this paper, we propose an implicit high-order compact (HOC) finite-difference scheme for solving the two-dimensional (2D) unsteady Navier–Stokes (N–S) equations on irregular geometries on orthogonal grids. Our scheme is second order accurate in time and fourth order accurate in space. It is used to solve three pertinent fluid flow problems, namely, the flow decayed by viscosity, the lid-driven square cavity and the flow in a constricted channel. It is seen to efficiently capture both transient and steady-state solutions of the N–S equations with Dirichlet as well as Neumann boundary conditions. Apart from including the good features of HOC schemes, our formulation has the added advantage of capturing transient viscous flows involving free and wall bounded shear layers which invariably contain spatial scale variations. Detailed comparison data produced by the scheme for all the three test cases are provided and compared with analytical as well as established numerical results. Excellent comparison is obtained in all the cases.

Keywords: HOC schemes; N–S equations; Constricted channel; Non-uniform grid; Reynolds number; Transient

1. Introduction

The governing equations representing the 2D unsteady incompressible viscous flow of a fluid are the N–S equations which, in non-dimensional primitive variable formulation, can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{Re}\nabla^2 u, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{Re}\nabla^2 v, \tag{3}$$

* Corresponding author. Tel.: +91 361 258 2615; fax: +91 361 258 2649.
E-mail addresses: swapan@iitg.ernet.in (S.K. Pandit), jiten@iitg.ernet.in (J.C. Kalita), durga@iitg.ernet.in (D.C. Dalal).

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where Re is the Reynolds number, P is the pressure and u, v are the velocity components along x- and y-directions respectively. Alternatively the streamfunction $\psi(x,y,t)$ and the vorticity $\zeta(x,y,t)$ can be defined as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}, \tag{4}$$

and

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \tag{5}$$

With these, the streamfunction–vorticity (ψ–ζ) form of the N–S Eqs. (1)–(3) can be written as

$$-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}=\zeta, \tag{6}$$

$$Re\frac{\partial\zeta}{\partial t}-\frac{\partial^2\zeta}{\partial x^2}-\frac{\partial^2\zeta}{\partial y^2}+uRe\frac{\partial\zeta}{\partial x}+vRe\frac{\partial\zeta}{\partial y}=0. \tag{7}$$

This ψ–ζ formulation has major advantages over the primitive variable form: firstly, it satisfies the continuity equation automatically and secondly, it decouples the pressure calculation from the velocity calculation. In the process, it also eliminates two computational difficulties, namely, finding (i) the correct boundary condition for pressure, and (ii) an explicit pressure equation satisfying the incompressibility constraint.

The past few decades have seen the development of many numerical schemes [3,4,7,9,12,16,19,20,22–24,26,27,29,32–34,37,40–43] to solve the N–S equations both in the primitive variables (1)–(3) as well as ψ–ζ ((6) and (7)) formulations. Some of these schemes utilize grid points located only directly adjacent to the node about which the differences are taken resulting in a formula involving a 9-point compact stencil for 2D cases. Off late, HOC finite-difference schemes based on 9-point compact stencil [15,16,19–21,29–32,34,40] for the computation of incompressible viscous flows are gaining popularity because of their advantages associated with high-order accuracy coupled with compact difference stencils. However, majority [15,16,20,21,29,31,32,34] of these HOC approaches on 9-point stencil are confined to steady flow calculations mostly on uniform space grids. As such these schemes could not fully exploit the advantages associated with using non-uniform grids, particularly that of mesh grading to resolve smaller scales in the regions of large gradients in the physical domain. Recently, Spotz and Carey [39], Zhang et al. [12], Kalita et al. [20], and Mancera [34] have developed some HOC schemes on non-uniform grids for the 2D convection–diffusion equations. Of these, the application of first two were limited to only linear problems whereas the third one, which used no transformation from the physical to the computational plane could accurately capture steady incompressible viscous flows governed by N–S equations, however, the last one based on Gupta's [15,16] idea is confined to steady flow calculations. Also, whenever there has been attempts to develop HOC scheme for the transient flows, they are confined invariably to uniform space grids [3,13,15,16,19,30]. Although, there exist solutions [2,27,36] of unsteady N–S equations using higher order schemes on non-uniform grids, these schemes could not be termed as compact in true sense; the stencil used in these schemes extends beyond one step length away from the point about which finite differences are taken.

In the present study we propose an HOC scheme based on 9-point compact stencil for the transient, spatially second order quasi-linear partial differential equation without the mixed-derivative term. The scheme which can be applied to both convection–diffusion and reaction–diffusion equations, can also be easily accommodated into solving equations of the N–S type with slight adjustment of the convection coefficients. It may be noted that application of almost all the schemes mentioned above was confined only to rectangular physical domains. The proposed scheme works equally efficiently on problems described on both rectangular as well as other curvilinear coordinate settings. It is implicit, second order accurate in time and fourth order accurate in space. It handles both Dirichlet and Neumann boundary conditions with ease. To validate the scheme, it is first applied to the problem of flow decayed by viscosity having analytical solutions and, then to the classical lid-driven square cavity problem. However, the power of the scheme is better realized when applied to capture the flow in a constricted channel on complex geometrical settings. We compare our numerical results with both analytical and established numerical results, and excellent match is obtained in all the cases.