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A balanced force refined level set grid method for two-phase flows on unstructured flow solver grids

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Abstract

A balanced force refined level set grid method for two-phase flows on structured and unstructured flow solver grids is presented. To accurately track the phase interface location, an auxiliary, high-resolution equidistant Cartesian grid is introduced. In conjunction with a dual-layer narrow band approach, this refined level set grid method allows for parallel, efficient grid convergence and error estimation studies of the interface tracking method. The Navier–Stokes equations are solved on an unstructured flow solver grid with a novel balanced force algorithm for level set methods based on the recently proposed method by Francois et al. [M.M. Francois, S.J. Cummins, E.D. Dendy, D.B. Kothe, J.M. Sicilian, M.W. Williams, A balanced-force algorithm for continuous and sharp interfacial surface tension models within a volume tracking framework, J. Comput. Phys. 213 (2006) 141–173] for volume of fluid methods on structured grids. To minimize spurious currents, a second order converging curvature evaluation technique for level set methods is presented. The results of several different test cases demonstrate the effectiveness of the proposed method, showing good mass conservation properties and second order converging spurious current magnitudes.

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1. Introduction

In liquid/gas flows, surface tension forces often play an important role. For example, during the atomization of liquid jets by coaxial fast-moving gas streams, the details of the formation of small-scale drops from aerodynamically stretched out ligaments is governed by capillary forces [32]. From a numerical point of view, surface tension poses a unique challenge since it is a singular force, active only at the location of the phase interface. In addition, the situation is further complicated by the fact that material properties, like density and viscosity, exhibit a discontinuity at the same location.

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One of the prerequisites for correctly treating surface tension forces is therefore the ability to locate the position of the phase interface accurately. To this end, several phase interface tracking schemes exist for fixed grid flow solvers, among them the marker method [56], the volume-of-fluid (VoF) method [19] and the level set method [52]. Each of these tracking methods has it is advantages and disadvantages, such that no clear gold-standard has emerged that is applicable to the wide range of possible two-phase flow phenomena. In this work, we will track the phase interface by a level set method. Level set methods are efficient, handle topology changes automatically, can directly solve for interfaces moving normal to themselves due to, for example, phase change and are easy to implement in parallel. Their main drawback is that liquid volume conservation is not guaranteed. Thus, hybrid methods have been proposed that make use of better volume conservation properties of an auxiliary interface tracking method to correct the level set representation of the interface. Among these are the particle level set method [13] using marker particles, or the CLSVOF method [51] and MCLS method [57] using the volume of fluid method. However, errors in the level set representation of the interface position are detected and corrected locally, thus potentially resulting in significant fluctuations in the higher derivatives of the level set scalar, i.e. the curvature or curvature derivative [21]. While de-localization techniques of the error correction can partly avoid this problem [11], both correction and de-localiaztion add an additional level of complexity to the scheme that is not always desired. Since the observed volume conservation error in level set methods is proportional to the employed grid resolution, an alternative approach is to employ fine enough grids to control the error. AMR techniques can be used to adaptively refine the grid in the vicinity of the interface, see for example [4,27,49,63], however these methods are usually complex in parallel applications and difficult to domain-decompose efficiently, unless block or patch refinement strategies are used [34,49].

In this paper, we propose to follow an alternative approach, termed refined level set grid (RLSG) method. In many technical applications of two-phase flows, like for example the atomization of liquid jets and sheets, the same high grid resolution is required virtually everywhere along the phase interface. Thus, a more practical approach is to uniformly refine the grid surrounding the phase interface. To avoid complex data structures like oct-trees, we propose to solve the level set equations on a separate, high resolution, equidistant Cartesian grid. The flow solver grid on which the two-phase Navier–Stokes equations are solved is independent of the level set grid and can be either structured or unstructured. To a certain extent the RLSG method is similar to the recently proposed narrow-band locally refined level set (NBLR-LS) approach by Gomez et al. [18]. However, the latter uses two-different grid levels of the same base grid and thus assumes a tight geometric coupling between the two grids. It is furthermore limited to Cartesian grids, whereas the RLSG method can deal with arbitrary unstructured finite volume flow solver meshes.

Unstructured grid methods for computing surface tension driven flows have been proposed in the past, mostly based upon finite element approaches [30,31,59,63]. These methods solve both the level set equation and the Navier–Stokes or Stokes equations on the same grid. However, since the RLSG method allows for the independent refinement of the interface tracking grid, grid convergence studies with respect to the interface tracking error can easily be performed. Furthermore, the approach allows for a separation of the error associated with the level set interface tracking scheme from the error associated with the solution of the Navier–Stokes equations since both grids can be independently refined enabling the calculation of separate error estimates.

Different strategies exist to discretize the surface tension force once the location of the phase interface is known. The most commonly used method is due to Brackbill et al. [7] called Continuum Surface Force (CSF). Here, the ideally singular surface tension force is spread into a narrow band surrounding the phase interface by the use of regularized delta functions. These can take the form of a discrete derivative of a Heaviside scalar (the volume fraction in VoF methods, or a Heaviside transform of the level set scalar [49]), or smoothed delta functions, like the popular cosine approximation due to Peskin [38] in level set methods . Especially in level set methods, the use of smoothed delta functions can be problematic, since convergence under grid refinement is only guaranteed for certain, not commonly employed delta function approximations [12].

The CSF method is prone to generating unphysical flows, so-called spurious currents, near the location of the phase interface when surface tension forces are present. In the canonical test cases of an equilibrium column and an equilibrium sphere, these velocity errors can grow unbounded very fast, if they are not artificially damped by introducing viscosity. The amplitude of the spurious currents when damped by viscosity is of the order of $u \sim 0.01\sigma/\mu$ for classical VoF and level set methods and $u \sim 10^{-5}\sigma/\mu$ for marker methods [43], where