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Reduced aliasing formulations of the convective terms within the Navier–Stokes equations for a compressible fluid

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Abstract

The effect on aliasing errors of different formulations describing the cubically nonlinear convective terms within the discretized Navier–Stokes equations is examined in the presence of a non-trivial density spectrum. Fourier analysis shows that the existing skew-symmetric forms of the convective term result in reduced aliasing errors relative to the conservation form. Several formulations of the convective term, including a new formulation proposed for cubically nonlinear terms, are tested in direct numerical simulation (DNS) of decaying compressible isotropic turbulence both in chemically inert (small density fluctuations) and reactive cases (large density fluctuations) and for different degrees of resolution. In the DNS of reactive turbulent flow, the new cubic skew-symmetric form gives the most accurate results, consistent with the spectral error analysis, and at the lowest cost. In marginally resolved DNS and LES (poorly resolved by definition) the new cubic skew-symmetric form represents a robust convective formulation which minimizes both aliasing and computational cost while also allowing a reduction in the use of computationally expensive high-order dissipative filters.

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Keywords: Navier–Stokes equations; Compressible flows; Convective operators; Skew-symmetric form; Aliasing

1. Introduction

Numerical simulations of the Navier–Stokes equations (NSE) for a compressible, reactive fluid are unavoidably plagued with some degree of error. Error sources include the truncation error associated with the integration and differentiation, round-off error, and error in the specification of the boundary conditions. Another important source of error is aliasing error [3,4] which arises during the differentiation of the product of two (or more) variables. This is particularly important in the evaluation of the convective terms of the Navier–Stokes equations. Aliasing error is present in pseudospectral, finite-difference, finite-volume, and finite-element approaches both in LES and DNS of turbulent flows. The aliasing phenomenon is often

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manifested by an unphysical growth of the spectral energy content of the integration variables at high wavenumbers. This energy growth at the high wavenumber end of the spectrum is accompanied by an equally artificial and accelerated decay of the energy content at the low wavenumber end of the spectrum. In situations where they are available, purely dissipative, high-order filters [15,19,23] may be used to remove the aliased energy accumulated at the highest wavenumbers of the spectrum. However, this technique does not compensate for the spectral energy loss at the low wavenumber modes. Therefore, it is generally prudent to select a numerical scheme which minimizes energy aliasing from the convection operator. In doing so, the need for high-order, dissipative filters will be minimized. Using minimally aliasing convection operators is particularly important in contexts where good filters are not readily available or there is little inherent dissipation in the numerical method. Low-order methods, with their high inherent dissipation, will likely see less benefit from better convection operators than higher-order methods. As there are other sources of high wavenumber information on the computational grid such as inexact derivative operators, high-order, dissipative filters are still an important tool in simulations of the NSE, when available.

Efforts to reduce aliasing error associated with the convective discretization of the NSE have their origin in constant density contexts [11] where the nonlinearities encountered in the convective terms are quadratic. Zang [29] performs tests to compare the four alternative formulations of the convective terms in the incompressible NSE: conservation/divergence, convection, rotation, and skew-symmetric. Subsequent to this, Horiuti and Itami [13], and Kravchenko and Moin [18] consider constant density flows while Blaisdell et al. [1,2], Chow and Moin [5], Ducros et al. [6], and Morinishi et al. [22] consider variable density flows. It is generally recognized that the aliasing error, for both constant and variable density flows, can be minimized by using a skew-symmetric formulation of the convective terms, however Boyd [3] importantly remarks that, if nonlinear interactions are so strong that non-trivial amounts of energy are being aliased, then the number of grid points should be increased in order to produce a meaningful solution. Other works on aliasing in simulations of an incompressible fluid may be found in the papers of Wilhelm and Kleiser [28], Lube and Olshanskii [21], Verstappen and Veldman [27], and Park and Mahesh [24].

In variable density flows, the convective terms are characterized by cubic nonlinearities and it is unclear whether one may simply recast a cubically nonlinear term using constructs specifically designed for quadratic nonlinearities [1,2,5,6,22]. Additionally, there is an inherent ambiguity in applying a skew-symmetric formulation constructed for quadratic nonlinearities to a cubically nonlinear term; three formulations are possible, two of which are mentioned in the open literature [1,9]. It should be stressed that if the density field is characterized by very small fluctuations and, consequently, it has no meaningful spectrum, then the variable density case will resemble the constant density case with quadratic nonlinearities. However, in compressible, multicomponent, reacting flows, the density will generally have a relatively broad spectrum. The objective of the paper is to explore a wide range of convective formulations that, to varying degrees, minimize aliasing error for quadratically and cubically nonlinear terms and to verify the robustness of these formulations in cases of a non-trivial density spectrum.

2. Background

The dimensional form of the Navier–Stokes equations for a compressible, multicomponent, reacting fluid is given by the following expressions:

$$\frac{\partial(\rho\mathbf{u}_i)}{\partial t} + \nabla_j\cdot(\rho\mathbf{u}_i\mathbf{u}_j) = \nabla_j\cdot(-p\delta_{ij} + \boldsymbol{\tau}_{ji}) + \rho\sum_{s=1}^{N_g} Y_s\mathbf{f}_{si} \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla_j\cdot(\rho\mathbf{u}_j) = 0 \tag{2}$$

$$\frac{\partial(\rho e_0)}{\partial t} + \nabla_j\cdot(\rho e_0\mathbf{u}_j) = \nabla_j\cdot(-p\mathbf{u}_j + \boldsymbol{\tau}_{ji}\cdot\mathbf{u}_i - \mathbf{q}_j) + \rho\mathbf{u}_j\cdot\sum_{s=1}^{N_g} Y_s\mathbf{f}_{sj} + \sum_{s=1}^{N_g}\mathbf{f}_{sj}\cdot\mathbf{J}_{sj} \tag{3}$$

$$\frac{\partial(\rho Y_s)}{\partial t} + \nabla_j\cdot(\rho Y_s\mathbf{u}_j) = -\nabla_j\cdot\mathbf{J}_{sj} + W_s\dot{\omega}_s \tag{4}$$