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Gas-kinetic scheme with discontinuous derivative for low speed flow computation

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ABSTRACT

The present paper concerns the improvement of the gas-kinetic scheme (GKS) for low speed flow computation. In the modified GKS scheme, the flow distributions with discontinuous derivatives are used as an initial condition at the cell interface for the flux evaluation. This discontinuity is determined by considering both the flow characteristic and grid's resolution. Compared with GKS method with a continuous slope for the flow variables at a cell interface, the new scheme is more robust and accurate. In the under resolved flow computation, the new scheme presents much less numerical oscillation. The extension of the current scheme to unstructured mesh is straightforward. To validate the method, both computations of 2D lid-driven cavity flow and 3D flow past a sphere are performed. The numerical results validate the current method.

1. Introduction

Gas-kinetic scheme based on approximate Boltzmann BGK model is a numerical method for compressible flows. It was first proposed by Xu in 1993 in his Ph.D. thesis [1], and got improved gradually afterwards. This kind of scheme targets on the high speed flow. Therefore, the van leer limiter for the initial data reconstruction is adopted to tackle physical shock wave which cannot be well resolved on the scale of computational cell. In the scheme, the evolution of particle velocity distribution function is governed by BGK equation, whose collision term is simplified as a relaxation term. The scheme couples both particle collision and translation simultaneously in the gas evolution process rather than separates them into individual process. Hence, GKS can simulate physical dissipation accurately and capture the Navier–Stokes solutions reasonably for low speed smooth flows.

In the original low speed GKS, the underline assumption at cell interface is that all flow variables have continuous first-order derivatives or slopes, which is used in numerical scheme. In 2001, Xu [2] used the Chapman–Enskog expansion to reconstruct the initial Navier–Stokes distribution function and indicated that the low speed scheme was a limiting case of a general GKS method. As a result, all computations related to the low speed flows took continuity assumption for flow variable distributions at a cell interface [3–6]. Su et al. [3] kept the energy equation in low speed scheme, and claimed that, if the Mach number is less than 0.15, the compressibility has little effect on incompressible simulation. The test cases in [3] were lid-driven cavity flow and back step flow. Xu and He [4] discarded the energy equation and employed isothermal model to simplify the scheme. He compared the GKS with LBM and presented new computations for the lid-driven cavity flow. Recently, Ni et al. [7] used a time-dependent gas distribution function to evaluate the conservative flow variables at cell

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interface and improved the efficiency of the scheme. But, in all above low speed GKS methods, the continuity assumption or a single slope for all flow variables at a cell interface are used. In fact, the schemes worked very well at low Reynolds number. However, at high Reynolds number, the previous numerical results showed some deficiencies. For examples, Su et al. [3] reported a steady solution of cavity flow at $Re = 10{,}000$, which should be an unsteady periodic solution according to recent progress [8,9]. Xu and He [4] observed artificial oscillation in the pressure distributions when Reynolds number exceeds 5000. In order to eliminate these deficiencies, we will show the effectiveness of introducing discontinuous derivatives for the flow variables at the cell interface for the low speed flow computations, and present a modified GKS method with improved accuracy and robustness, especially in the cases where the cell resolution is not enough to resolve the physical flow structures.

This paper is organized as follows. Section 2 clarifies the merit and weakness of original method for low speed simulation and presents a new low speed GKS method. Sections 3 and 4 show the simulation results for 2D lid-driven cavity flow at high Reynolds number and 3D flow passing over a sphere, where the simulation results are compared with experimental measurements and other classical results. The last section is the discussion and conclusion.

2. Low speed gas-kinetic scheme

2.1. Basic processes of GKS

There are two key stages in gas-kinetic scheme. The first stage is called reconstruction stage. It translates discrete macroscopic variables into particle distribution functions in (x,y,u,v,ξ) space, where (x,y) are spatial coordinates, (u,v) are particle velocities in corresponding direction and ξ denotes the internal freedom of motion. The second stage is gas evolution stage. It includes solving governing equations (BGK equation) at the cell interface and updating macroscopic variables inside each control volume. Fig. 1 shows the basic processes of this kind of method.

The governing equation is the BGK equation:

$$\frac{\partial f}{\partial t} + \vec{u} \cdot \frac{\partial f}{\partial \vec{x}} = \frac{g - f}{\tau}, \tag{1}$$

where f is particle velocity distribution function of particle velocity $\vec{u}$ and internal freedom of motion ξ, and g is the corresponding local equilibrium state which is a Maxwell distribution,

$$g = \rho \left(\frac{\lambda}{\pi}\right)^{\frac{N+3}{2}} e^{-\lambda((u-U)^2+(v-V)^2+\xi^2)}, \tag{2}$$

where N is the degree of internal freedom and $\lambda = \frac{\rho}{2p}$. Here ρ and p are density and pressure, respectively.

2.2. Original gas-kinetic scheme

2.2.1. High speed method

In high speed compressible flow problem, since real shock layer thickness is much thinner than the mesh size, in order to describe the flow variables on large grid scale, the initial reconstruction of flow variables will involve discontinuities. In this process, nonlinear limiter must be used to capture the jump (Fig. 2). Therefore, the scheme is of appropriate dissipation and less artificial oscillations around shock waves. Otherwise, continuous reconstruction cannot represent the natural discontinuity and generate annoying artificial oscillation. In hypersonic flow problem, a modification of particle collision time [10] and incorporating multiple temperature modes [11] must be adopted to model real gas effects.

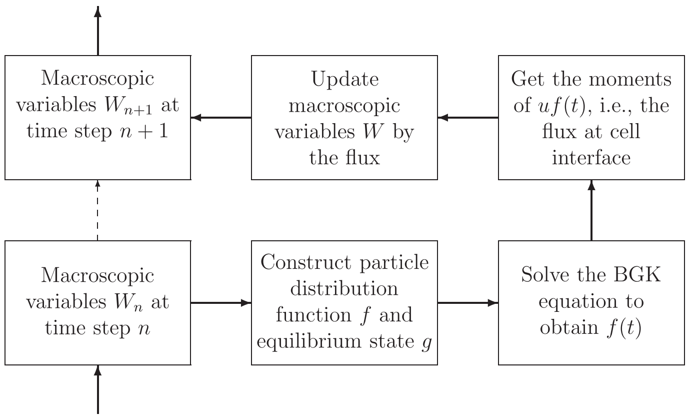

Fig. 1. Basic processes of GKS.