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Analysis of iterated ADI-FDTD schemes for Maxwell curl equations

B.D. Welfert

Department of Mathematics and Statistics, Arizona State University, Tempe, AZ 85287-1804, United States

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Abstract

The convergence of the iterative ADI-FDTD method proposed by Wang et al. [S. Wang, F. Teixeira, J. Chen, An iterative ADI-FDTD with reduced splitting error, IEEE Microwave Wireless Comp. Lett. 15 (2005) 1531–1533] towards the classical implicit Crank–Nicolson scheme when applied to Maxwell curl equations, and the accuracy, stability, and dispersion properties of the resulting iterated schemes are investigated. The iterated schemes are shown both mathematically and numerically to be unconditionally stable for 2D wave problems, in agreement with numerical experiments conducted in [S. Wang, F. Teixeira, J. Chen, An iterative ADI-FDTD with reduced splitting error, IEEE Microwave Wireless Comp. Lett. 15 (2005) 1531–1533]. However these schemes lose their unconditional stability when applied to full 3D wave problems where TE and TM modes do not decouple, as illustrated by numerical experiments in a PEC box.

Keywords: Finite difference time domain; Alternate direction implicit scheme; Fixed-point iteration; Iterated scheme; von Neumann stability; Unconditional stability; Dispersion relation

1. Introduction

The alternating-direct ion-implicit (ADI) finite-difference-time-domain method [26,12] is a popular scheme for solving the three-dimensional Maxwell curl equations. The ADI scheme is a compromise between standard explicit schemes such as the popular Yee scheme [21], which is efficient but unstable for larger time steps (due to CFL restrictions), and fully implicit schemes such as Crank–Nicolson (CN), which is unconditionally stable but inefficient (a 3D system must be solved at each time-step). On the other hand, the ADI scheme combines efficiency (requiring the solution of one-dimensional systems only at each time-step), with unconditional stability, and has been successfully applied to a variety of wave propagation and scattering problems, in particular in low frequency bioelectromagnetics.

Both CN and ADI schemes are second-order accurate (in the classical, non-stiff, sense) in time and in space. However, the ADI scheme lacks isotropy in dispersion properties and has been observed to have inferior

E-mail address: welfert@asu.edu.
URL: http://math.asu.edu/~bdw.

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accuracy, compared to the CN scheme, in regions with large field gradients, e.g. around singularities associated with corners or near-field sources [8].

A variety of modifications of the ADI scheme designed to circumvent these problems have recently emerged.

- Richardson extrapolation and deferred correction ideas were considered in [10,6] to improve the accuracy in the ADI solution. The extrapolation method combines two second-order ADI solutions obtained with different step sizes into a single fourth-order approximation, while the deferred correction method uses an estimate of the local truncation error of the ADI scheme, obtained from a standard ADI solution, as an explicit source term in a second ADI run. These ideas can be iterated to provide additional accuracy, although successive extrapolations/corrections may have an undesirable effect on stability [10].
- Efforts to correct the ADI solution "on the fly", i.e., at the time-step level, include ideas based on extrapolation [1], symmetrization [20], and iterative correction [19]. Numerical results provided in the references seem to indicate that these strategies may be beneficial in improving the accuracy. However none of them considers the effect of modifications on stability, and numerical experiments only address two-dimensional TE or TM wave applications.

In this study we examine, both analytically and numerically, the iterative correction approach introduced in [19]. This approach defines a sequence of iterated ADI schemes using a fixed point (FP) iteration on the CN equations preconditioned with ADI. The resulting iterated schemes were tested in [19] on a two-dimensional TE wave only (an unspecified extrapolated version was mentioned). Further experiments on TE wave problems with a variable number of fixed-point iterations according to local spatial requirements were conducted in [18]. In particular, we investigate: (a) the convergence of the fixed-point iteration towards the CN scheme; and (b) the accuracy, stability and dispersion properties of the iterated schemes obtained for a fixed number of fixed-point iterations.

In Section 2 we set up the Maxwell curl system in Fourier space used in the subsequent accuracy and stability analysis. Known accuracy, stability and dispersion properties of the CN, ADI, and the Yee schemes [21] are summarized in Section 3. The iterated schemes are analyzed in Section 4. We show that the sequence of iterated schemes does converge to the CN scheme, that each scheme resulting from a fixed number of fixed-point iterations is unconditionally stable when applied to two-dimensional TE or TM wave problems (as in the numerical experiments conducted in [18,19]), but are unstable when applied to full three-dimensional wave problems with coupled TE and TM modes, even at low CFL numbers. Our conclusions are supported by a complete Fourier analysis of the 2D case, as well as numerical experiments in a three dimensional PEC box, reported in Section 5.

2. Maxwell system in Fourier space

We consider the (scaled) 3D Maxwell system

$$\partial_t u = \mathscr{A} u + \mathscr{B} u \tag{1}$$

in free space, where $u = [E,H]^{\mathrm{T}}$ and

$$\mathcal{A} = \left[\begin{array}{ccc|ccc} & & & 0 & 0 & \partial_y \\ & & & \partial_z & 0 & 0 \\ & & & 0 & \partial_x & 0 \\ \hline 0 & \partial_z & 0 & & & \\ 0 & 0 & \partial_x & & & \\ \partial_y & 0 & 0 & & & \end{array}\right], \quad \mathcal{B} = \left[\begin{array}{ccc|ccc} & & & 0 & -\partial_z & 0 \\ & & & 0 & 0 & -\partial_x \\ & & & -\partial_y & 0 & 0 \\ \hline 0 & 0 & -\partial_y & & & \\ -\partial_z & 0 & 0 & & & \\ 0 & -\partial_x & 0 & & & \end{array}\right]. \tag{2}$$