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Runge–Kutta discontinuous Galerkin methods for compressible two-medium flow simulations: One-dimensional case

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Abstract

The Runge–Kutta discontinuous Galerkin (RKDG) method for solving hyperbolic conservation laws is a high order finite element method, which utilizes the useful features from high resolution finite volume schemes, such as the exact or approximate Riemann solvers, TVD Runge–Kutta time discretizations, and limiters. In this paper, we investigate using the RKDG finite element method for compressible two-medium flow simulation with conservative treatment of the moving material interfaces. Numerical results for both gas–gas and gas–water flows in one-dimension are provided to demonstrate the characteristic behavior of this approach.

AMS: 65M60; 65M99; 35L65

Keywords: Runge–Kutta discontinuous Galerkin method; WENO scheme; Ghost fluid method; Approximate Riemann problem solver

1. Introduction

In this paper, we investigate using the Runge–Kutta discontinuous Galerkin (RKDG) finite element method for compressible two-medium flow simulation in one-dimensional case.

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The first discontinuous Galerkin (DG) method was introduced in 1973 by Reed and Hill [28], in the framework of neutron transport (steady-state linear hyperbolic equations). A major development of the DG method was then carried out by Cockburn et al. in a series of papers [11,10,9,8,12], in which they established a framework to easily solve the *nonlinear* time dependent hyperbolic conservation laws:

$$\begin{cases} u_t + \nabla \cdot f(u) = 0 \\ u(x,0) = u_0(x) \end{cases} \tag{1.1}$$

using an explicit, nonlinearly stable high order Runge–Kutta time discretizations [30] and DG discretization in space with exact or approximate Riemann solvers as interface fluxes and limiters such as the total variation bounded (TVB) limiters [29] or weighted essential non-oscillatory (WENO) type limiters [26,27], to achieve non-oscillatory properties for strong shocks. These schemes are termed RKDG methods. RKDG methods have been widely applied and performed very well to solve the single-medium compressible flow.

A relatively dominant difficulty for simulating compressible two-medium flow is the treatment of moving material interfaces and their vicinities. Nonphysical oscillations usually occur in the vicinity of the material interface when a well-established numerical method for single-medium flow is directly applied to multi-medium flow. In the literature there are some methods developed to overcome this difficulty [19,17,1,7,21,2,4].

The ghost fluid method (GFM) developed by Fedkiw et al. [13] has provided an attractive and flexible way to treat the two-medium flow. The main appealing features of the GFM are its simplicity, easy extension to multi-dimensions and maintenance of a sharp interface without smearing. The GFM makes the interface "invisible" during computations by defining ghost cells and ghost fluids, and the computations are then carried out as for a single-medium manner via solving two respective single-medium GFM Riemann problems. As such, its extension to multi-dimensions becomes fairly straightforward. Since only single-fluid flux formulations are required to make GFM workable, the GFM is easily employed for two fluids of vastly different types such as a compressible-incompressible or viscous-inviscid two-fluid flow [5]. Variants of the original GFM [14] and other applications can also be found in [2,18]. Recently, efforts have also been made to develop a conservative GFM as found in [15,3].

On the other hand, it is precisely the manner of treatment of the single medium across the interface in the GFM that may cause numerical inaccuracy when there is a strong shock wave interacting with the interface [23]; this is especially so if such wave interaction with the interface is not taken into account properly in the definition of the ghost fluid state. This situation arises because the pattern of shock refraction at a material interface and the resultant interfacial status depend highly on material properties on both sides of the interface. As such, reasonable ghost fluid states have to be formulated to take into account the influence of both material properties and wave interaction with the interface. This has led to the development of a modified GFM (MGFM) with a predicted ghost fluid status by Liu et al. [23] via implicitly solving two non-linear characteristic equations interacting and applicable at the interface [21,22]. In fact, it has been found that Conditions have to be satisfied for the ghost fluid state in order that the two GFM Riemann provides the correct solution in the respective real fluids [20]. Those techniques developed in [21,22] will also be used to calculate the flow interface state in the multi-medium RKDG algorithm as proposed in this work.

In general, algorithms proposed for solving two-medium compressible flow consist of two parts. One is the method for solving the single-medium flow and the other is to treat the interface of the two fluids. In [13,5,14], the 3rd order ENO methods are used together with the GFM for treating the interface, while the Godunov-type or MUSCL (monotone upwind schemes for conservation laws) are used in [1,2,21,22,4,23,20].

In this paper our intent is to investigate using the RKDG finite element method for multi-medium flow simulations in one-dimensional case. Similar to the above-mentioned algorithms developed for multi-medium compressible flows, the present method to be proposed also consists of two parts. One is the usual RKDG algorithm applicable for the flow field away from the material interface; the other is the newly developed DG technique for treating the moving interface. In Section 2, we first briefly review the usual RKDG method over a fixed and regular mesh system, and then we describe in detail the extension of the DG to treat the moving material interfaces conservatively. Extensive numerical results are presented in Section 3 to illustrate the characteristic behavior of the RKDG method presented in Section 2. Concluding remarks are given in Section 4.