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Journal of Computational Physics 226 (2007) 1645–1667
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

A global finite-difference semi-Lagrangian model for the adiabatic primitive equations

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Received 3 August 2006; received in revised form 24 May 2007; accepted 7 June 2007
Available online 22 June 2007

Abstract

We develop a global semi-implicit semi-Lagrangian model for the atmospheric adiabatic primitive equations discretized through finite-differences. The model formulation includes a new semi-Lagrangian treatment of the continuity equation and a spatially averaged Eulerian handling of the orography. These techniques contribute to the accuracy and efficiency of the scheme. The semi-Lagrangian discretization makes the integration method very stable; we can carry out integrations with time-steps which by far exceed the CFL time-step limitations of Eulerian schemes. We carry out several numerical experiments, showing that good accuracy is achieved even when we triple the time-steps. Our numerical experiments also demonstrate the computational efficiency of the method; we can run 10 days simulations at fine resolutions in a few hours on a personal computer.
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PACS: 92.60.Wc; 02.70.Bf; 02.60.Cb; Ams 65M06

Keywords: Semi-Lagrangian; Finite-differences; Global weather models; Primitive equations; Multigrid

1. Introduction

Semi-Lagrangian methods have been widely adopted in global numerical weather prediction models, in conjunction with spectral [17,19], finite-elements [7] and finite-differences [6,12,10] discretizations. The main advantage of semi-Lagrangian schemes is the fact that they are not limited by CFL-type restrictions in the choice of time-step sizes, leading, in principle, to computationally more efficient schemes. There are, however, several aspects to be considered in the development of an efficient numerical method for the global primitive equations.

In the present article, we develop a three-time-level semi-implicit semi-Lagrangian method, based on a finite-difference discretization, for the three-dimensional primitive equations. We employ a uniform latitude–longitude Arakawa C-grid on the sphere and a pressure based σ vertical coordinate. The semi-Lagrangian scheme is formulated in vector form and is handled as in [16]. We introduce a new

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doi:10.1016/j.jcp.2007.06.011

semi-Lagrangian treatment of the continuity equation, employing a mean (vertically averaged) wind in the advection of surface pressure. The treatment of this equation becomes essentially two-dimensional and therefore cheaper than other approaches as in [6] or [17]. We compare our formulation with those other approaches and get similar accuracy in the numerical experiments at lower computational costs. We have also adapted the spatially averaged Eulerian treatment of orography proposed in [18] to our finite-difference model. Our experiments show the advantages of this formulation in the presence of steep orography, especially if large time-steps are employed.

For the validation of the model we have employed the test case suggested by Polvani et al. [13]. We have run several tests, at different resolutions, and we have obtained good agreement with the results given in [13].

We present numerical results demonstrating the stability of the scheme, even with time-steps which exceed by far the CFL constraints of Eulerian schemes. At every time-step of the model, the semi-implicit part of the discretization leads to a three-dimensional scalar equation to be solved. This equation is decomposed into a set of two-dimensional Helmholtz-type equations, through the use of the precomputed eigenvectors of the matrix of the vertical structure. We show that this is possible and that the resulting equations are elliptic, by proving that the vertical structure matrix has a complete set of eigenvectors and that all its eigenvalues are positive. The Helmholtz-type equations are solved efficiently with the help of a multigrid solver adapted from [2]. The computational complexity of the scheme varies linearly with an increase in the number of grid-points in each model layer, in contrast with a higher complexity of spectral methods which employ Legendre transforms. Only the process of decoupling the three-dimensional scalar equation associated with the semi-implicit discretization has a higher complexity, being quadratic on the number of layers of the model. The final efficiency of the model is very good. We are able to carry out 10 days forecasts at a spatial resolution of less than one degree (0.9375°), with 28 vertical layers, in about three and half hours on a personal computer (with a 3.2 GHz Pentium 4, 2 MB cache and 3 GB of memory).

The paper is arranged as follows. In Section 2, we present the complete description of the model. A series of numerical experiments focusing on the model validation, the multigrid performance, the variations of treatment of the orography and of the continuity equation, the use of large time-steps and its effect on accuracy and results on the model performance are presented in Section 3. We finish with some conclusions. We also point out that the present work is a step towards a variable resolution model for the primitive equations, in continuation to our work with shallow-water models [4].

2. The model formulation

The dry, adiabatic, primitive equations build the core of most global weather models. They are given by the momentum equations

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} + f\mathbf{k}\times\mathbf{V} + \nabla_{\mathrm{H}}\Phi + RT\nabla_{\mathrm{H}}\ln p_{\mathrm{s}} = 0, \tag{1}$$

the thermodynamic equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\kappa T}{\sigma p_{\mathrm{s}}}\omega = \kappa T\left(\frac{\dot{\sigma}}{\sigma} + \frac{\partial \ln p_{\mathrm{s}}}{\partial t} + \mathbf{V}\cdot\nabla_{\mathrm{H}}\ln p_{\mathrm{s}}\right) \tag{2}$$

and the continuity equation

$$\frac{\partial \ln p_{\mathrm{s}}}{\partial t} + \mathbf{V}\cdot\nabla_{\mathrm{H}}\ln p_{\mathrm{s}} + D + \frac{\partial\dot{\sigma}}{\partial\sigma} = 0, \tag{3}$$

where we have adopted $\sigma = p/p_{\mathrm{s}}$ as the vertical independent coordinate (p is pressure and p_{s} it's value on the Earth's surface). The 'horizontal' coordinates are latitude and longitude (λ,θ), covering the whole sphere. The prognostic variables of the model are the horizontal wind field $\mathbf{V} = (u, v)$, the temperature T and the surface pressure p_{s}. The other variables in the equations, which can be obtained diagnostically, are the geopotential height Φ, obeying the hydrostatic equation:

$$\sigma\frac{\partial\Phi}{\partial\sigma} + RT = 0 \tag{4}$$

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