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A 3D spectral anelastic hydrodynamic code for shearing, stratified flows

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Abstract

We have developed a three-dimensional (3D) spectral hydrodynamic code to study vortex dynamics in rotating, shearing, stratified systems (e.g., the atmosphere of gas giant planets, protoplanetary disks around newly forming protostars). The time-independent background state is stably stratified in the vertical direction and has a unidirectional linear shear flow aligned with one horizontal axis. Superposed on this background state is an unsteady, subsonic flow that is evolved with the Euler equations subject to the anelastic approximation to filter acoustic phenomena. A Fourier–Fourier basis in a set of quasi-Lagrangian coordinates that advect with the background shear is used for spectral expansions in the two horizontal directions. For the vertical direction, two different sets of basis functions have been implemented: (1) Chebyshev polynomials on a truncated, finite domain, and (2) rational Chebyshev functions on an infinite domain. Use of this latter set is equivalent to transforming the infinite domain to a finite one with a cotangent mapping, and using cosine and sine expansions in the mapped coordinate. The nonlinear advection terms are time-integrated explicitly, the pressure/enthalpy terms are integrated semi-implicitly, and the Coriolis force and buoyancy terms are treated semi-analytically. We show that internal gravity waves can be damped by adding new terms to the Euler equations. The code exhibits excellent parallel performance with the message passing interface (MPI). As a demonstration of the code, we simulate the merger of two 3D vortices in the midplane of a protoplanetary disk.

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1. Introduction

Three of Jupiter's notable features are: rapid rotation (spin period of just under 10 h); an atmosphere striped with a large number of alternating zones and belts corresponding to strongly shearing east–west winds (hundreds of m/s); and many long-lived, coherent vortices, the most prominent being the great red spot (GRS). Of course, these three characteristics – rotation, shear, and vortices – are all dynamically linked, so much so that it is often claimed that the presence of the first two implies the likely existence of the third [31,32]. Protoplanetary disks (the disks of gas and dust in orbit around newly-forming protostars) also have rapid rotation and intense shear, which has inspired proposals that such disks should also be populated with long-lived, coherent storms [1,5–8]. These vortices may play two critical roles in star and planet formation: (1) In cool, non-magnetized disks, vortices may transport angular momentum radially outward so that mass can continue to accrete onto the growing protostar, and (2) vortices are very efficient at capturing and concentrating dust particles, which may help in the formation of planetesimals, the basic ''building blocks'' of planets.

Motivated by these geophysical and astrophysical problems, we have developed a three-dimensional (3D) spectral hydrodynamic code that employs specially tailored algorithms to handle the computational challenges due to rapid rotation, intense shear, and strong stratification. In subsonic flow, short-wavelength acoustic waves have periods that are much shorter than the characteristic timescale of the large-scale advective motions. In numerical simulations, the timestep for an explicit algorithm must be short enough to temporally resolve these fast waves (i.e., the Courant–Friedrich–Lewy, or CFL, condition), which may be inefficient for calculating the evolution of the slower, large-scale flow for long integration times. One strategy is to filter sound waves from the fluid equations (''sound-proofing'') so that the timestep will be limited by the longer advective timescale. The anelastic approximation does this by replacing the full continuity equation with the kinematic constraint that the mass flux be divergence-free. This approximation still allows for the effects of density stratification (e.g., buoyancy in the vertical momentum equation, pressure–volume work in the energy/entropy equation) and has been employed extensively in the study of deep, subsonic convection in planetary atmospheres [4,24,37] and stars [20,34]. In [8], we re-derived the anelastic approximation in the context of protoplanetary disks. Stratified media support the propagation of internal gravity waves. As these waves travel from high density to low density regions, their amplitudes can grow to large values (so as to conserve energy flux). If the density contrast is large, velocity and thermodynamic fluctuations can become sufficiently large so as to invalidate the anelastic approximation and/or violate the CFL condition. We have developed a technique based on ''negative feedback'' to damp these waves when they propagate into low-density gas which has very little inertia.

We compute the evolution of the anelastic equations with a spectral method. The basic philosophy of spectral methods is to approximate any function of interest with a finite sum of basis functions multiplied by spectral coefficients [11,13,23,30]. A partial differential equation (PDE) in space and time is reduced to a coupled set of ordinary differential equations (ODE) for the time evolution of the spectral coefficients. The chief advantage of spectral methods over finite-difference methods is accuracy per degrees of freedom (e.g., number of spectral modes or number of grid points). In one dimension, the global error (e.g., L_2 norm) for a finite-difference method with N grid points scales as $(1/N)^p$, where p is the (fixed) order of the method, whereas for a spectral method with N spectral modes, the error scales as $(1/N)^N$. Thus, to get the same level of accuracy, spectral methods generally require far fewer degrees of freedom. This advantage is even more pronounced in 3D problems requiring high resolution.

Because of the linear background shear, the fluid equations depend explicitly on the cross-stream coordinate (i.e., non-autonomous), making it problematic to apply periodic boundary conditions in this direction. The equations can be made autonomous in the horizontal directions by transforming to a set of Lagrangian shearing coordinates [22,33,39]. Features in the flow that are advected by the shear appear quasi-stationary in the shearing coordinates, allowing for larger timesteps to be taken in the numerical integration. Because the background state generally depends on the vertical coordinate, we do not impose periodicity in the vertical direction. We have implemented the code with two different sets of vertical basis functions: (1) Chebyshev polynomials on a truncated, finite domain, and (2) rational Chebyshev functions on an infinite domain. Use of this latter set is equivalent to transforming the infinite domain to a finite one with a cotangent mapping, and using cosine and sine expansions in the mapped coordinate [11,12].