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Cost-effectiveness of fully implicit moving mesh adaptation: A practical investigation in 1D

Giovanni Lapenta *, Luis Chacón

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87544, USA

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Abstract

The cost-effectiveness of moving mesh adaptation is studied in a number of 1D tests. We propose a method that is based on two established modern techniques. First, we use a moving mesh approach based on the classic equidistribution method. Second, we discretize the model equations for grid and physics using a conservative finite volume method and we solve the resulting equations with a preconditioned inexact Newton–Krylov method.

Using these state of the art methods, we consider the question of whether a real improvement in performance can be achieved using adaptive grids. We consider rigorous metrics of the accuracy and cost of a numerical solution on uniform and adaptive grids. For a number of classic but challenging problems we demonstrate that indeed adaptive grids can lead to a great improvement in cost-effectiveness.

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Keywords: Adaptive grids; Implicit methods; Grid generation; Newton–Krylov; Fully implicit methods; Moving mesh

1. Introduction

Adaptive grids are becoming an ever more common tool for high performance scientific computing. We focus here on the type of adaptation achieved by moving a constant number of points according to appropriate rules, an approach termed moving mesh adaptation (MMA).

A great literature body exists on the subject of moving mesh adaptation, and we refer the reader to the excellent textbooks on the subject [1,2]. The approach we consider here is based specifically on retaining a finite volume approach but allowing the grid to evolve in time according to a grid evolution equation obtained from minimization principles. The approach originates from the seminal papers by Brackbill and Saltzmann [3] and by Winslow [4].

In the present paper we consider the fundamental question in the application of MMA. Is it worth the effort? The literature is very rich and considerable results have been obtained in designing MMA approaches

* Corresponding author. Tel.: +1 505 667 5493; fax: +1 505 665 3107.
E-mail address: lapenta@lanl.gov (G. Lapenta).

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that provide grids that can indeed present the desired properties [5–10]. But the question of whether once the adaptive grids are used the simulations are actually more cost-effective remains largely unanswered.

Some efforts are notable in the literature in trying to ascertain the cost-effectiveness of grid adaptation. For example, a study of geophysical flow solvers based on MMA [11] has shown that the use of adaptive grids within an explicit time discretization approach is only marginally advantageous in 1D and becomes more cost-effective only in 2D where the gains obtained by adaptation in each direction are compounded. This is a crucial consideration, any gains obtained in 1D can theoretically be squared in 2D and cubed in 3D. Of course, in practice, complexities added by 2D and 3D reduce the theoretical gain, but the fact remains, as demonstrated in [11], that 1D adaptation presents the fundamental challenge in proving cost-effectiveness of grid adaptation. However, unlike Ref. [11], the present study relies on implicit time discretization and significant gains are demonstrated even in 1D.

We intend here to revisit the question of cost-effectiveness of adaptive grids in 1D problems bringing to bear the latest developments in modern numerical analysis. Three issues are key in affecting cost-effectiveness of grid adaptation.

First is the formulation of the moving grid equations. In 1D the problem is benign, as error minimization leads to error equidistribution and to a rigorous and simple minimization procedure [2]. However, in 2D and 3D the problem is more serious. In the present paper the focus is on 1D problems, but the approach followed here can be extended to higher dimensionality [12] using the classic approach by Brackbill–Saltzmann–Winslow [3,4]. A crucial feature of our approach is the formulation of the physics equations in a conservative form. The independent variables of the physics equations are changed from the physical to the logical space and the equations are rewritten in the logical space in a fully conservative form [2]. In a separate paper we consider the extension of the implicit MMA methods to 2D problems [12].

Second is the solution algorithm for the MMA method. Here we bring a new development. The moving mesh equations and the physics equations, derived by discretizing the problem under investigation on a moving grid, form a tightly coupled system of algebraic non-linear equations. Traditionally, the coupling is broken, the physics and grid equations being solved separately in a lagged time-splitting approach. Each time step is composed of two alternating steps: the physics equations are solved on the current grid, the grid equations are then solved using new information from the solution of the physics equations. However, in presence of sharp fronts or other moving features, breaking such coupling can lead to grid lagging with respect of the physics equations, with adaptation resulting behind rather than on the moving feature.

We avoid breaking the coupling and solve the full non-linear set of physics and grid equation using the preconditioned Newton–Krylov (NK) approach [13]. Fully non-linear Newton methods have been used before in different frameworks for MMA such as in the moving finite element approach [14], in finite difference approaches [7,5,8], particularly for radiation-hydrodynamics [6,9].

Third ingredient in a cost-effective grid adaptation is good error indicators. Among the three ingredients this appears, to our subjective reading of the literature, as the less mature of the three. Keeping our helm fixed on the goal of probing the cost-effectiveness of the MMA method, we decided not to rely on unproven experimental error indicators. We focus instead on proven albeit heuristic measures widely used in the literature, such as the arc-length or the curvature error indicators [15].

The rest of the paper is organized as follows. Section 2 describes the moving mesh formalism used. Section 3 describes a model advection–diffusion equations used in the investigation as well as its finite volume discretization. Section 4 describes our novel solution procedure based on modern preconditioned NK methods. Section 5 reports the results of our cost-effectiveness analysis for a suite of classic single and multiple scale problems. Conclusions are drawn in Section 6.

2. Moving mesh adaptation equations

Adaptation based on a fixed number of points and a fixed connectivity among the points can be obtained using the concept of mapping between a logical uniform space and the actual physical space. A large literature body is available on this approach, e.g. Ref. [2]. In multi-dimensions, the problem can only be tackled at a complex mathematical level. In 1D, instead, a simple derivation suffices. Below, we restrict the attention to 1D systems, but using methods that have been extended to 2D [3,16,17,12].