Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Physics 219 (2006) 144–162
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

A fully implicit numerical method for single-fluid resistive magnetohydrodynamics

Daniel R. Reynolds [a,*], Ravi Samtaney [b], Carol S. Woodward [c]

[a] Department of Mathematics, University of California at San Diego, 9500 Gilman Drive, Dept 0112, La Jolla, San Diego, CA 92093-0112, United States
[b] Princeton Plasma Physics Lab, MS 26, P.O. Box 451, Princeton, NJ 08543, United States
[c] Lawrence Livermore National Lab, P.O. Box 808, L-551, Livermore, CA 94551, United States

Received 13 May 2005; received in revised form 4 March 2006; accepted 18 March 2006
Available online 11 May 2006

Abstract

We present a nonlinearly implicit, conservative numerical method for integration of the single-fluid resistive MHD equations. The method uses a high-order spatial discretization that preserves the solenoidal property of the magnetic field. The fully coupled PDE system is solved implicitly in time, providing for increased interaction between physical processes as well as additional stability over explicit-time methods. A high-order adaptive time integration is employed, which in many cases enables time steps ranging from one to two orders of magnitude larger than those constrained by the explicit CFL condition. We apply the solution method to illustrative examples relevant to stiff magnetic fusion processes which challenge the efficiency of explicit methods. We provide computational evidence showing that for such problems the method is comparably accurate with explicit-time simulations, while providing a significant runtime improvement due to its increased temporal stability.

Keywords: Newton–Krylov; Implicit couplings; Resistive magnetohydrodynamics

1. Introduction

1.1. Motivation

The design of next-generation magnetic fusion devices requires increased understanding of nonlinear macroscopic stability, reconnection processes and refueling approaches for burning plasmas. Due to the high cost of conducting physical experiments of these processes in magnetic fusion devices, researchers are increasingly turning to computational simulation as a tool for such scientific investigation. However, it is well known that

* Corresponding author. Tel.: +1 858 534 5862; fax: +1 858 534 5273.
E-mail addresses: drreynolds@ucsd.edu (D.R. Reynolds), samtaney@pppl.gov (R. Samtaney), cswoodward@llnl.gov (C.S. Woodward).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2006.03.022

the numerical modeling of magnetic-confinement fusion systems is one of the most challenging problems in contemporary computational physics. This is a result of many factors, including the complexity of models that accurately represent burning plasmas, as well as the resolution of the large range of spatio-temporal scales at which significant physical processes occur. A key result of temporal stiffness is that traditional explicit methods used for solution to such models may experience prohibitively small time step restrictions compared to the dynamical scales of macroscopic stability and plasma fueling.

In this work, we propose a fully implicit numerical approach for solving the single-fluid, resistive magnetohydrodynamic equations which constitute one of the relevant models describing burning plasma processes at the device scale. We root our implicit numerical methods in a Backwards Differentiation Formula–Newton–Krylov solution framework. There are many attractive qualities of such numerical techniques for these problems, including their resolution of nonlinear couplings between the disparately evolving internal physical processes and their increased temporal stability compared to traditional numerical solution methods for these problems.

A true description of plasma motion must rely on kinetic equations for each plasma species. As this approach is too costly for simulation of full magnetic fusion devices, a fluid description of the plasma is often used. This description is obtained by taking velocity moments of the kinetic equations describing a plasma under certain closure assumptions and the assumptions of large collisionality (see [1] for details). Magnetohydrodynamics, or MHD, is the term given to a single fluid description of a plasma in which a single velocity and pressure describe both the electrons and ions. This approximation is distinguished from *two-fluid MHD* in which electrons and ions retain separate pressures and velocities. The simplest MHD model is that of *ideal MHD*, which ignores the diffusion terms arising from collisions, assuming that these effects are negligible compared with other terms. When these diffusion terms are retained, the mathematical model is referred to as *single-fluid resistive MHD*, which is the primary focus of this paper. While single-fluid resistive MHD may be considered to be one of the simplest models used to describe plasma dynamics, it is nonetheless rich in mathematical structure and has been successfully employed to simulate physics at the device-scale [2,3]. We note that there have been a number of recent developments in the literature that are based on related physical models incorporating simplifications and/or incorporation of additional physical processes. An oft-used approximation of the MHD system in the presence of a strong magnetic field is to constrain the plasma compressibility in the direction perpendicular to the field. This asymptotic expansion results in simplified sets of modeling equations, and is generally referred to as *reduced MHD*. Additional processes that have been modeled are two-fluid effects including the Hall term and electron pressure gradients, under the umbrella of *extended MHD* [4].

In particular, we are interested in resistive MHD modeling of tokamaks, magnetic fusion devices in a toroidal confinement configuration having a strong background toroidal magnetic field. In the MHD modeling of tokamaks (and other confinement configurations), the numerical difficulties stem from: (1) a wide range of space scales, (2) a wide range of time scales, and (3) a large anisotropy induced by the background magnetic field. The presence of a large background field and toroidal geometry separates the effective speeds of the MHD waves into three branches, each with characteristic wave speeds that differ from one another by approximately an order of magnitude. Thus, if the characteristic transit times of the fast magnetosonic wave, the shear Alfvén wave, and the slow magnetosonic wave are denoted τ_F, τ_A and τ_S, these satisfy $\tau_F \ll \tau_A \ll \tau_S$. We are specifically interested in problems where the physical processes under study occur on the τ_A time-scale, or slower. Thus, for explicit methods whose time step is restricted by the CFL condition to be at the τ_F time-scale, the calculation will require an excessive number of time steps for resolution of the relevant physical processes. Our motivation to develop a Newton–Krylov technique for implicit integration of the resistive MHD equations stems from the desire to follow the dynamics relevant to these physical problems of interest and not the τ_F time scale restriction. Thus, this work is mainly concerned with addressing the second issue mentioned above, i.e., the wide range of time scales. We also note that Newton–Krylov techniques have been demonstrably successful in the implicit solution of similarly stiff physical systems, such as the Navier–Stokes equations, radiation hydrodynamics and a variety of other applications [5–7].

This paper is organized as follows. In the next subsection we briefly review previous work on different time approaches to MHD modeling. Section 2 introduces the single-fluid resistive MHD equations, and our numerical methods are described in Section 3. We then present results on a suite of test problems designed to verify