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An algebraic variational multiscale–multigrid method for large-eddy simulation of turbulent variable-density flow at low Mach number

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ABSTRACT

An algebraic variational multiscale–multigrid method is proposed for large-eddy simulation of turbulent variable-density flow at low Mach number. Scale-separating operators generated by level-transfer operators from plain aggregation algebraic multigrid methods enable the application of modeling terms to selected scale groups (here, the smaller of the resolved scales) in a purely algebraic way. Thus, for scale separation, no additional discretization besides the basic one is required, in contrast to earlier approaches based on geometric multigrid methods. The proposed method is thoroughly validated via three numerical test cases of increasing complexity: a Rayleigh–Taylor instability, turbulent channel flow with a heated and a cooled wall, and turbulent flow past a backward-facing step with heating. Results obtained with the algebraic variational multiscale–multigrid method are compared to results obtained with residual-based variational multiscale methods as well as reference results from direct numerical simulation, experiments and LES published elsewhere. Particularly, mean and various second-order velocity and temperature results obtained for turbulent channel flow with a heated and a cooled wall indicate the higher prediction quality achievable when adding a small-scale subgrid-viscosity term within the algebraic multigrid framework instead of residual-based terms accounting for the subgrid-scale part of the non-linear convective term.

1. Introduction

Large-eddy simulation (LES) of turbulent flow aims at resolving the larger flow structures and modeling the effect of the smaller ones. The variational multiscale approach to LES (VMLES) as originally proposed in [1] and later addressed, e.g., in [2] further separates the resolved scales into larger and smaller ones; see, e.g., [3] for a review and several references therein. The other distinguishing feature of VMLES compared to the traditional LES approach is to be found in using variational projection instead of filtering for scale separation. The consideration of three scale groups (i.e., large resolved, small resolved, and unresolved scales) was already used before in the context of the dynamic modeling procedure proposed in [4]. It is based on a scale-similarity hypothesis (with respect to smaller resolved and unresolved scales) earlier exploited in the scale-similarity model [5].

LES has successfully been applied to both incompressible (see, e.g., [6]) and compressible (see, e.g., [7]) turbulent flow. More rarely, applications of LES to turbulent variable-density flow at low Mach number are encountered in literature. This

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is despite the importance of the problems mathematically described by this set of equations. In particular, problems of combustion are usually mathematically described by a variable-density formulation of the Navier–Stokes equations for low-speed flows, see, e.g., [8,9]. A rigorous derivation of the equation system at low Mach number with a view on combustion problems can be found, e.g., in [10]. Methods for LES of reactive and/or non-reactive turbulent low-Mach-number flow are proposed, for instance, based on finite-volume approaches in [11–15] and based on a finite-difference approach in [16]. In most studies, the version of the dynamic Smagorinsky model in [17], which extended the original proposal in [4] for incompressible flow to the compressible case and scalar transport, is used as subgrid-scale model. For the scalar transport equation, a subgrid diffusivity is used which is defined by the ratio of the subgrid viscosity used in the momentum equation and a turbulent Prandtl number as earlier proposed by Erlebacher et al. see, e.g., [18]. An overview on traditional subgrid-scale models for turbulent compressible flows can also be found in [19].

All of the aforementioned LES studies were used within finite-volume- or finite-difference-based computational environments. Finite element methods (FEM) for non-reactive low Mach number flow using inf-sup stable elements are described, e.g., in [20–22]. Residual-based variational multiscale methods or stabilized FEM were recently proposed for such problems in [23,24]. Stabilized FEM for reactive flow had been addressed before by Hauke and Valiño (see [25]), by Shadid and various co-workers at Sandia National Laboratories (see, e.g., [26], and references therein) as well as by Braack and co-workers (see, e.g., [27], and references therein). However, all aforementioned publications merely addressed *laminar* low-Mach-number flow situations. In particular, to the best of our knowledge, there have not yet been any studies on FEM for (non-reactive) variable-density *turbulent* flows at low Mach number published.

In a predecessor study to the present one, [28], we developed a residual-based variational multiscale method for low-Mach number flow suitable for simulating laminar via transitional to turbulent flow regimes. That approach extends the respective method proposed in [29] for turbulent incompressible flow to the low-Mach number case and is based on the general framework of the variational multiscale method (see, e.g., [30]). It has similarities with the methods used in [23,24]. However, we already observed considerably improved results in the context of turbulent incompressible flow when including a small-scale subgrid-viscosity model in the sense of a VMLES. The present work will show that such an improvement is also achievable for variable-density flow by developing a formulation which features such a small-scale subgrid-viscosity model in the sense of a VMLES. For a method to be used for a VMLES, the crucial aspect is the way large and small resolved scales are separated.

The framework of an algebraic variational multiscale–multigrid method (AVM^3) was originally proposed in [31] and applied to convection-dominated convection–diffusion problems. It was further developed and extended for application to turbulent flow in the form of LES in [32]. The scale separation is based on level-transfer operators arising in plain aggregation algebraic multigrid (PA-AMG); see, e.g., [33]. Though conceptually different, PA-AMG is closely related to volume-agglomeration multigrid methods (see, e.g., [34,35]), which were preferably developed for finite-volume discretizations of hyperbolic problems. A scale separation inspired by a volume-agglomeration method as proposed in [34] was used in [36] within a VMLES. Geometric multigrid approaches to LES had already been proposed in [37] and later in [38]; see also the recent review in [39]. Those methods were not derived using the framework of the VMLES. A geometric multigrid approach to VMLES was later developed in [40]. Compared to those geometric multigrid procedures, the present algebraic multigrid method obviates the often challenging generation of additional meshes besides the basic one.

The present study proposes the AVM^3 for LES of turbulent variable-density flow at low Mach number. The study is organized as follows. In Section 2, the variable-density equation system at low Mach number in two alternative formulations is given. Afterwards, a residual-based variational multiscale formulation is derived in Section 3. This section serves two purposes. On the one hand, the reader is familiarized with the method the AVM^3 is compared to. On the other hand, some of the terms introduced in the residual-based variational multiscale formulation are also employed for the AVM^3, which is then provided in Section 5, after a brief presentation of the time-integration and solution procedures in Section 4. The AVM^3 is then validated for three numerical examples, a Rayleigh–Taylor instability, turbulent channel flow with a heated and a cooled wall and turbulent flow past a backward-facing step with heating, in Section 6. Results obtained with the AVM^3 are compared to results obtained with residual-based variational multiscale methods as well as reference results from direct numerical simulation (DNS), experiments and LES published elsewhere. Conclusions from this study are drawn in Section 7.

2. Two formulations of the variable-density equations at low Mach number

2.1. Temperature formulation

Conservation equations for mass, momentum and energy in the domain Ω are given in convective form as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p_{\text{hyd}} - \nabla \cdot (2\mu \varepsilon'(\mathbf{u})) = \rho \mathbf{g}, \tag{2}$$

$$\rho \frac{\partial T}{\partial t} + \rho \mathbf{u} \cdot \nabla T - \nabla \cdot \left(\frac{\lambda}{c_p} \nabla T \right) = \frac{1}{c_p} \left[\frac{dp_{\text{the}}}{dt} + Q \right], \tag{3}$$