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Well-balanced finite volume schemes for pollutant transport by shallow water equations on unstructured meshes

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Abstract

Pollutant transport by shallow water flows on non-flat topography is presented and numerically solved using a finite volume scheme. The method uses unstructured meshes, incorporates upwinded numerical fluxes and slope limiters to provide sharp resolution of steep bathymetric gradients that may form in the approximate solution. The scheme is non-oscillatory and possesses conservation property that conserves the pollutant mass during the transport process. Numerical results are presented for three test examples which demonstrate the accuracy and robustness of the scheme and its applicability in predicting pollutant transport by shallow water flows. In this paper, we also apply the developed scheme for a pollutant transport event in the Strait of Gibraltar. The scheme is efficient, robust and may be used for practical pollutant transport phenomena.

Keywords: Shallow water equations; Pollutant transport; Finite volume method; Unstructured grids; Strait of Gibraltar

1. Introduction

During the last years the increase of pollution in rivers, lagoons and coastal regions has attracted much interest in numerical methods for the prediction of its transport and dispersion. In many situations, this pollution problem has detriment impact on the ecology and environment and may cause potential risk on the human health and local economy. Efficient and reliable estimates of damages on the water quality due to pollution could play essential role in establishing control strategy for environmental protection. Introduction and utilization of such measures are impossible without knowledge of various processes such as formation of water flows and transport of pollutants. The mathematical models and computer softwares could be very helpful to understand the dynamics of both, water flow and pollutant transport. In this respect mathematical

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modeling of water flows and the processes of transport-diffusion of pollutants could play a major role in establishing scientifically justified and practically reasonable programs for long-term measures for a rational use of water resources, reduction of pollutant discharge from particular sources, estimation of the impact in the environment of possible technological improvements, development of methods and monitoring facilities, prediction and quality management of the environment, etc. The success of the mathematical and computer methods in solving practical problems depends on the convenience of the models and the quality of the software used for the simulation of real processes.

Clearly, the process of pollutant transport is determined by the characteristics of the fluid flow and the properties of the pollutant. Thus, dynamics of the fluid and dynamics of the pollutant must be studied using a mathematical model made of two different but dependent model variables: (i) an hydrodynamic variable defining the dynamics of the flow, and (ii) a concentration variable defining the transport of the pollutant. In the current work, the hydrodynamic model is based on a two-dimensional shallow water equations while, an advection–diffusion equation is used for the pollutant transport. For environmental flows, the shallow water system is a suitable model for adequately describing significant hydraulic processes. The different characteristics of pollutants require an appropriate model to describe their dynamics, nevertheless for a wide class of dispersed substances the standard advection–diffusion equation can be used. The interaction between the two processes gives rise to an hyperbolic system of conservation laws with source terms.

The accurate solution of shallow water flows is of major importance in most of pollutant transport predictions. In many practical applications, the shallow water equations have to be solved on non-flat and rough beds, and also on topographic structures covering different spatial scales. Thus, the treatment of topography and friction source terms is of major importance in these applications. It is well known that shallow water equations on non-flat topography have steady-state solutions in which the flux gradients are non-zero but exactly balanced by the source terms. This well-balanced concept is also known by exact conservation property (C-property), compare [13,11,38]. Computational techniques using finite difference, finite element and finite volume methods have been extensively reported in the literature. Although widely applied to shallow water equations, the finite difference technique has the major drawback that it does not guarantee strict conservation of mass and momentum. Furthermore, the necessity of including process across a range of spatial scales means that techniques capable of operating on unstructured meshes will be more appropriate than those such as the finite difference methods which rely on structured and often regular meshes. The finite element method has been used with irregular meshes of triangular or quadrilateral elements to model shallow water flows [17,15]. However, the finite element method can experience difficulty when both subcritical and supercritical flows are encountered [3], and may produce solutions with local mass conservation errors in some implementations [19]. The finite volume method is therefore adopted in the present work. For a comprehensive review of recent developments in finite volume methods for shallow water equations we refer to [37].

Various numerical methods developed for general systems of hyperbolic conservation laws have been applied to the shallow water equations. For instance, most shock-capturing finite volume schemes for shallow water equations are based on approximate Riemann solvers which have been originally designed for hyperbolic systems without accounting for source terms such as bed slopes and friction losses. Therefore, most of these schemes suffer from numerical instability and may produce nonphysical oscillations mainly because dicretizations of the flux and source terms are not well-balanced in their reconstruction. The well-established Roe's scheme [33] has been modified by Bermúdez and Vázquez [11] to treat source terms. This method has been improved by Vázquez [38] for general one-dimensional channel flows. However, for practical applications, this method may become computationally demanding due to its treatment of the source terms. Alcrudo and Garcia-Navarro [5] have presented a Godunov-type scheme for numerical solution of shallow water equations. Alcrudo and Benkhaldoun [4] have developed exact solutions for the Riemann problem at the interface with a sudden variation in the topography. The main idea in their approach was to define the bottom level such that a sudden variation in the topography occurs at the interface of two cells. LeVeque [24] proposed a Riemann solver inside a cell for balancing the source terms and the flux gradients. However, the extension of this scheme for unstructured meshes is not trivial. Numerical methods based on surface gradient techniques have also been applied to shallow water equations by Zhou et al. [42]. The TVD-MacCormak scheme has been used by Ming-Heng [28] to solve water flows in variable bed topography. A different approach based on local hydrostatic reconstructions have been studied by