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On the Huygens absorbing boundary conditions for electromagnetics

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Abstract

A new absorbing boundary condition (ABC) is presented for the solution of Maxwell equations in unbounded spaces. Called the Huygens ABC, this condition is a generalization of two previously published ABCs, namely the multiple absorbing surfaces (MAS) and the re-radiating boundary condition (rRBC). The properties of the Huygens ABC are derived theoretically in continuous spaces and in the finite-difference (FDTD) discretized space. A solution is proposed to render the Huygens ABC effective for the absorption of evanescent waves. Numerical experiments with the FDTD method show that the effectiveness of the Huygens ABC is close to that of the PML ABC in some realistic problems of numerical electromagnetics. It is also shown in the paper that a combination of the Huygens ABC with the PML ABC is very well suited to the solution of some particular problems.

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1. Introduction

In recent years, two novel absorbing boundary conditions (ABC) appeared in the literature, for use in numerical electromagnetics with the finite-difference time-domain (FDTD) method. These ABCs are the multiple absorbing surfaces (MAS) condition [1], and the re-radiating boundary condition (rRBC) [2,3]. They were developed independently and formulated in slightly different manners, but both rely on the same basic principle that consists of cancelling the outgoing field leaving the domain by means of equivalent currents that radiate a field equal in magnitude and opposite in sign to the field to be cancelled.

The principle of the MAS and rRBC is then simple and attractive. Unfortunately, some difficulties arise as implementing them, because the outgoing field on the surface where the equivalent currents are impressed,

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called the Huygens surface [4], cannot be known rigorously. An operator is used to obtain an estimate of the equivalent currents. From this, the cancellation is not perfect, resulting in a certain amount of reflection from the proposed ABCs.

In this paper, we revisit this kind of ABCs. We consider more general ABCs, called Huygens ABCs (HABCs), that hold as special cases the MAS and the rRBC. We derive theoretical properties of the Huygens ABCs in continuous spaces as well as in the discrete FDTD space. This permits results and observations in [1–3] to be interpreted. Especially, we show that the residual field radiated outside the Huygens surface is the time derivative of the outgoing field, while the field reflected towards the inner domain is integrated on time as it passes back through the Huygens surface. We also show that the overall reflection from a HABC is equal, rigorously, to the reflection from the ABC formed with the operator used to estimate the equivalent currents. This is true for both traveling and evanescent waves. From this, methods [1–3], and more generally any HABC, cannot be viewed as novel ABCs. They are only alternative implementations of ABCs based on use of operators, for example alternative implementations of Higdon operator ABCs [5,6]. In consequence, ABCs [1–3] suffer from the same drawbacks as the operator ABCs. The most important is the strong reflection of the evanescent waves present in many problems of electromagnetics solved with numerical methods. This is verified in this paper with a FDTD experiment.

In the last part of the paper we show that Huygens ABCs are of valuable interest in some problems of numerical electromagnetics, despite of their equivalence to previously known operator ABCs. As shown in [1–3], high-order HABCs can be easily implemented by juxtaposing several one-order HABCs, without the stability problems faced when implementing high-order operator ABCs. However, the operators used in [1–3] are only designed to absorb traveling waves, they reflect in totality evanescent waves, so that the domain of application of the MAS [1] and rRBC [2,3] is probably narrow. We suggest two alternative ways to apply Huygens ABCs in an effective manner in realistic applications of numerical electromagnetics. The first way relies on the introduction of operators designed to absorb evanescent waves. By means of HABCs they can be easily combined with such traditional operators as Higdon operators. This permits effective absorption of both traveling and evanescent waves present in many problems, like using the complex frequency shifted (CFS) PML ABC [7,8]. This is illustrated with a waveguide problem using the FDTD method. The second suggested use of HABCs relies on their easy combination with the PML ABC. This is of interest in certain problems where both traveling waves and evanescent waves are present at low frequency. Then, a HABC placed in front of a CFS PML permits the traveling waves to be absorbed at low frequency, where the CFS PML is transparent to these waves [7,8]. This is discussed in detail and illustrated in the paper with a FDTD experiment.

2. Principle of the absorbing boundary condition based on use of a Huygens surface

In electromagnetics, the equivalence theorem states that the field produced within a given part of space by sources located outside this part can be reproduced by impressing the following electric and magnetic current densities upon the surface separating the two parts:

$$\vec{J}_s = \vec{n} \times \vec{H}_i \tag{1a}$$

$$\vec{K}_s = -\vec{n} \times \vec{E}_i \tag{1b}$$

where $\boldsymbol{n}$ is the unit vector normal to the surface, oriented in the direction opposite to the sources, and $\boldsymbol{E}_i$ and $\boldsymbol{H}_i$ are the fields that would exist upon the surface if the sources were present. The surface where equivalent currents are set is called a Huygens surface [4]. The equivalent currents radiate no field in the part of space where the sources are present. This permits incident waves to be enforced in finite methods [4] and is a requirement for achieving reflectionless Huygens ABCs.

An important remark can be done about the equivalent currents (1). If the orientation of the unit vector were reversed, that is the unit vector oriented towards the sources, the sign of the equivalent currents would be also reversed, so that the radiated field would be opposite to the field radiated by the sources. In a different context [9], such a Huygens surface whose unit vector in (1) is opposite to its physical orientation has been called an anti-Huygens surface. In the context of the Huygens ABC, a fundamental consequence of this