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Journal of Computational Physics 214 (2006) 284–298
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

Quasineutral particle simulation technique for whistlers ☆

Martin Lampe [a,*], Glenn Joyce [b], Wallace M. Manheimer [c], Anatoly Streltsov [c], Gurudas Ganguli [a]

[a] *Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375-5346, United States*
[b] *George Mason University, Fairfax, VA, United States*
[c] *Icarus Research, Inc., Bethesda, MD, United States*

Received 3 March 2005; received in revised form 19 September 2005; accepted 21 September 2005
Available online 9 November 2005

Abstract

We present a new hybrid fluid/PIC simulation scheme for whistlers, which eliminates both the speed-of-light time scale and the electron plasma oscillation time scale, and concentrates simulation resources on the resonant parts of electron phase space that control whistler evolution. The code runs with time steps on the order of the electron gyrofrequency, with extremely accurate energy conservation and numerical stability. Examples are shown of application to whistler instability growth and saturation, and ducting of whistlers in density channels.

Keywords: Plasma simulation; Particle simulation; Particle-in-cell simulation; Whistlers; Magnetospheric plasma

1. Introduction

Whistlers [1,2] are right-hand polarized electromagnetic waves that are carried by the electrons in a plasma, with frequency in the range below the electron cyclotron frequency $\Omega \equiv eB_0/mc$ but above the lower-hybrid frequency. Here, B_0 is the ambient magnetic field and m is the electron mass. Whistlers are slow waves, i.e., the phase velocity $\omega/k \ll c$, and therefore the source for the wave magnetic field is overwhelmingly the electron current, rather than the displacement current. Furthermore, in situations of interest the electron plasma frequency $\omega_p \equiv (4\pi n_0 e^2/m)^{1/2}$ is typically much larger than Ω, and thus certainly very large compared to ω. Here n_0 is the ambient electron density. The waves are thus quasineutral, i.e., wave perturbations $n - n_0$ are small compared to n_0 itself, even if the wave amplitude is large. (It is important to note that this does not mean that the electrostatic field is negligible; a small perturbation to n can lead to a very large electrostatic field. Ion

☆ Supported by Office of Naval Research.
* Corresponding author. Tel.: +1 202 767 4041; fax: +1 202 767 1607.
E-mail address: lampe@nrl.navy.mil (M. Lampe).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2005.09.015

sound waves, for example, are in essence quasineutral electrostatic waves.) Numerical simulation schemes involving the use of a straightforward electromagnetic field solver are thus extremely inefficient, since the time steps must be small enough to resolve the Courant condition for speed c, and must also be small enough to resolve the plasma frequency time scale. Both of these time scales are much smaller than the wave period. In addition, *particle simulation* schemes based on a full electromagnetic field solver are particularly susceptible to noise, since the charge density $\rho(\mathbf{x},t) \equiv -e[n(\mathbf{x},t) - n_0(\mathbf{x})]$ is obtained from small perturbations to the finite number of simulation particles in a given region, and the electrostatic component of the electric field in essence derives from $\rho(\mathbf{x},t)$ through Poisson's equation.

Traditionally, the Darwin model [3] has been used to address these issues that arise in numerical simulation of slow electromagnetic waves. In this approach, the *solenoidal* component of the displacement current (i.e., essentially the transverse component, for a linear plane wave) is neglected in Maxwell's equations. The *irrotational* component of the displacement current cannot be neglected, since it is essentially the time derivative of the electrostatic field, i.e., the longitudinal electric field in a linear plane wave. The separation of the solenoidal component of the displacement current is difficult, and it enormously complicates the algorithmic schemes as well as significantly increasing the running time of Darwin codes. Quasineutrality is a separate issue that has also been addressed within the Darwin scheme [3,4], but it leads to additional complications.

Another approach which has been used [5,6] to study the nonlinear evolution of whistlers is to begin with equations relating the slowly varying quantities ψ and α that characterize the long-time evolution of resonant electrons. Here, ψ is the difference in phase between the gyro motion of an electron and the magnetic field of the wave, and α is the pitch angle of the electron. This approach permits the use of long-time steps, but it can only be used for the case of a single discrete wave, as the definition of ψ inherently depends on there being well-defined (but possibly slowly varying) values of k and ω for the wave.

In the present paper, we present a new, efficient and simple approach which is not restricted to the case of a single mode, and which takes advantage of the characteristics of whistlers (and other low-frequency electromagnetic waves). Specifically, a whistler is carried primarily by the large number of cool non-resonant electrons, and it is quite accurate to represent these electrons as a cold fluid [1,2]. However, the whistler may be damped, or driven unstable, by a smaller class of fast electrons that are in cyclotron resonance with the wave. These electrons also engage in complex nonlinear behaviors such as trapping and stochastic diffusion that control the evolution of large-amplitude waves and can result in nonlinear effects such as instability saturation and triggered emission. In our hybrid scheme, the energetic electrons of interest are followed using the particle-in-cell (PIC) technique, while the bulk of the electrons are represented as a cold fluid, which can be advanced numerically orders of magnitude faster. Because of the presence of the cold fluid, we are able, within a fully self-consistent scheme, to eliminate the *entire* displacement current from Maxwell's equations (not just the solenoidal part), and to determine the electrostatic field directly from the requirement of quasineutrality, rather than by requiring that Poisson's equation be satisfied. Speed-of-light phenomena cannot occur since there is no displacement current, and electron plasma oscillations cannot occur since quasineutrality is enforced. It is therefore not necessary to resolve the ω_p time scale. The result is a code which we call HEMPIC (hybrid electromagnetic PIC) that runs with time steps typically $\sim 1/4\Omega$. At present we are running a 2-D Cartesian version of HEMPIC, but there is no reason why the scheme cannot be 2-D cylindrical or fully 3-D. Also at present, HEMPIC follows only the motion of electrons and treats the ions as an immobile neutralizing background, but the approach could be extended to include ion kinetics, in order to study, e.g. ion cyclotron waves.

2. Basic equations: cold fluid only

To introduce the novel features of our scheme in the clearest way, we shall begin by discussing a simplified model in which the plasma is represented entirely as a cold electron fluid, with an immobile neutralizing ion background. We shall use the subscript c to denote cold-electron quantities. In a later section, we generalize the model to include PIC simulation electrons in addition to the cold electron fluid, and we shall then use the subscript p to denote particle-electron quantities.