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An adaptive GRP scheme for compressible fluid flows

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ABSTRACT

This paper presents a second-order accurate adaptive generalized Riemann problem (GRP) scheme for one and two dimensional compressible fluid flows. The current scheme consists of two independent parts: Mesh redistribution and PDE evolution. The first part is an iterative procedure. In each iteration, mesh points are first redistributed, and then a conservative interpolation formula is used to calculate the cell-averages and the slopes of conservative variables on the resulting new mesh. The second part is to evolve the compressible fluid flows on a fixed nonuniform mesh with the Eulerian GRP scheme, which is directly extended to two-dimensional arbitrary quadrilateral meshes. Several numerical examples show that the current adaptive GRP scheme does not only improve the resolution as well as accuracy of numerical solutions with a few mesh points, but also reduces possible errors or oscillations effectively.

1. Introduction

The generalized Riemann problem (GRP) scheme, as an analytic second order accurate extension of the Godunov scheme, was originally developed for one-dimensional (1D) system of an unsteady and inviscid flows [1,3]. The basic idea of the GRP scheme consists of replacing the exact solution by a piecewise linear function and analytically solving a generalized Riemann problem at each cell interface to yield numerical fluxes. Recently a direct Eulerian GRP scheme was presented in [4,5,20–22], aiming at getting rid of the auxiliary Lagrangian scheme, which was essential in [1], and solving the one-dimensional (1D) generalized Riemann problem directly in the Eulerian coordinates by employing the regularity property of the Riemann invariants.

The direct Eulerian GRP scheme is efficient and robust in capturing hydrodynamic singularities (shocks and contact discontinuities) for most cases. In the meantime it also inherits drawbacks from many Godunov-type schemes, such as the instability of stationary shocks and start-up errors in a single shock wave of the one-dimensional compressible fluid flows. For two-dimensional cases, the GRP scheme was extended by employing the Strang splitting method in [5], but it had to be restricted on uniform rectangular meshes. The purpose of this paper is to develop one-dimensional and two-dimensional adaptive GRP schemes by combining the Eulerian GRP scheme [5] with the adaptive moving mesh method [31]. With such efforts, the afore mentioned drawbacks or restrictions are totally overcome.

Our present adaptive GRP scheme consists of two independent parts: Evolution of PDEs with the GRP scheme on an arbitrary quadrangular mesh and the mesh redistribution with the Gauss–Seidel iteration method. In this context one key

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ingredient is to extend the Eulerian GRP scheme over arbitrary quadrangular meshes. The reason is that the adaptive moving mesh method can locally cluster or spread out mesh points according to the solution variations, and that this method will totally break the uniform mesh distribution. Note that it makes sense in its own to develop a two-dimensional or multi-dimensional GRP scheme on arbitrary meshes. Our approach of the two-dimensional GRP scheme on arbitrary quadrangle meshes is to integrate the conservation laws over an hexahedral control volume in the space time domain, and then use the average of conservative variables (mass, momentum, and energy) to replace their integrals on the top and bottom faces. Numerical fluxes on four lateral faces are approximated by replacing true solutions with the centroid point values, which are analytically obtained by solving the associated generalized Riemann problem in the time direction. This generalized Riemann problem is defined in the outward normal direction of each quadrangle boundary.

Another key ingredient in the present scheme is to define the primitive slope after every step of the mesh iteration distribution, since the primitive slope cannot be directly obtained on the resulting new mesh. The conservative interpolation developed in [31] is used to solve this problem. Some efficient monitor functions are discussed and summarized in numerical experiments. From the numerical results in Section 5, we can see that the adaptive GRP scheme will not only improve the resolution as well as the accuracy of numerical solutions with much fewer meshes, but also effectively reduce possible errors (oscillations), which may be present in many Godunov-type schemes for the compressible fluid flows [17]. That is to say, the present adaptive GRP scheme provides an efficient and robust way to achieve very accurate solutions, thus it would be useful in practical applications.

This paper is organized as follows. In Section 2, the direct Eulerian GRP scheme for two-dimensional planar compressible fluid flows is reviewed and extended to arbitrary quadrangular meshes for general two-dimensional cases. In Section 3, the adaptive moving mesh method is illustrated, in particular, the method how to remap the primitive slope. Section 4 shows the algorithm of the adaptive GRP scheme. Numerical experiments are carried out in Section 5 to display the performance of this scheme. A final conclusion is given in Section 6.

2. The GRP scheme in one and two-dimensions

In this section we first review the GRP scheme for two-dimensional (2D) planar compressible fluid flows, and then extend it for general two-dimensional cases on arbitrary quadrangular meshes. This serves to develop our adaptive GRP scheme, although it has its own significance.

2.1. Review of 2D planar GRP scheme

The following system is used to describe 2D planar compressible fluid flows,

$$\tfrac{\partial U}{\partial t} + \tfrac{\partial F(U)}{\partial x} = 0, \quad U = \begin{pmatrix} \rho \\ \rho u \\ \rho v E \end{pmatrix}, \quad F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ uE + pu \end{pmatrix}, \tag{2.1}$$

where ρ, e are the density and the internal energy, respectively, $p = p(\rho, e)$ is the pressure, u is the velocity along the x-direction and v is the velocity perpendicular to the x-direction, $E = \rho e + \rho\frac{u^2+v^2}{2}$ is the total energy.

Denote spatial grid points with $\{x_j; j = 1, \ldots, J\}$, and interface points $x_{j+\frac{1}{2}} = (x_j + x_{j+1})/2$. Define the cells $C_j = \left[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}\right]$ with length $\Delta x_j = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}$. Let U_j^n be the average value of U over the cell C_j at time $t_n = n\Delta t$, and assume that the data at time $t = t_n$ are piecewise linear with a slope σ_j^n,

$$U(x, t_n) = U_j^n + \sigma_j^n (x - x_j), \qquad x \in \left(x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}\right). \tag{2.2}$$

Then the GRP scheme consists of the following three steps to define a pair of unknown vectors $(U_j^{n+1}, \sigma_j^{n+1})$.

Step 1. Given (2.2), calculate mid-point values $U_{j+\frac{1}{2}}^{n+\frac{1}{2}}$ approximately with the formula

$$U_{j+\frac{1}{2}}^{n+\frac{1}{2}} = U_{j+\frac{1}{2}}^{n} + \frac{\Delta t}{2}\left(\frac{\partial U}{\partial t}\right)_{j+\frac{1}{2}}^{n}, \tag{2.3}$$

where $U_{j+\frac{1}{2}}^n$ and $\left(\frac{\partial U}{\partial t}\right)_{j+\frac{1}{2}}^n$ are defined via solving the local generalized Riemann problem (GRP) at $\left(x_{j+\frac{1}{2}}, t_n\right)$, and specified in Section (2.2).

Step 2. Evaluate the interior cell averages by using the updating formula

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x_j}\left(F\left(U_{j+\frac{1}{2}}^{n+\frac{1}{2}}\right) - F\left(U_{j-\frac{1}{2}}^{n+\frac{1}{2}}\right)\right), \quad j = 1, \ldots, J. \tag{2.4}$$

Step 3. Update the slope σ_j^{n+1} by the following procedure,