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Journal of Computational Physics 212 (2006) 1–5

JOURNAL OF COMPUTATIONAL PHYSICS

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Short note

Moore's law and the Saffman–Taylor instability

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Received 28 December 2004; received in revised form 18 May 2005; accepted 29 June 2005
Available online 19 August 2005

Ten years ago Hou, Lowengrub and Shelley [4] published a state-of-the-art boundary integral simulation of a classical viscous fingering problem, the Saffman–Taylor instability [12]. In terms of complexity and level of detail, those computations [4] are still among the most ramified and accurately computed interfacial instability patterns that have appeared in the literature. Since 1994, the computational power of a standard workstation has increased a hundredfold as predicted by Moore's law [7]. The purpose of this Note is to consider Moore's law and its consequences in computational science, and in particular, its impact on studying the Saffman–Taylor instability. We illustrate Moore's law and fast algorithms in action by presenting the worlds largest viscous fingering simulation to date.

Viscous fingering is one of the fundamental interfacial instabilities in fluid dynamics: Perturbations to an expanding circular air bubble displacing a viscous fluid in a thin gap flow device become unstable, resulting in intricate densely branched interfacial patterns. The viscous fingering problem is governed by the three-dimensional incompressible Navier–Stokes equations with moving free boundaries at the fluid/air interface. Without simplification, this problem is computationally intractable and will remain so for the foreseeable future. The simulations in this Note build upon decades of advances in mathematical modeling and numerical methods: (i) The fluid dynamics in a thin gap is reduced from the three-dimensional Navier–Stokes equations by asymptotic analysis to a Darcy's law [12]

$$\mathbf{u} = -\frac{b^2}{12\mu}\nabla p, \quad \nabla \cdot \mathbf{u} = 0 \tag{1}$$

for a two-dimensional velocity and pressure field (b is the gap thickness and μ is the viscosity); (ii) The three-dimensional boundary conditions at the free-surface are approximated at Γ to leading order [9] by a two-dimensional Young–Laplace boundary condition $[p] = -\gamma\kappa$ relating the pressure, the surface tension γ, and curvature κ, and by the kinematic condition $\partial \mathbf{x}/\partial t = \mathbf{u}$; (iii) The pressure satisfies a Laplace equation

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doi:10.1016/j.jcp.2005.06.022

which is written in a boundary integral formulation [2]; (iv) The integral equation is solved iteratively using GMRES [11] without forming the matrix explicitly; (v) The integral operator [2] is evaluated at each iteration using the fast multipole method (FMM) [3] in O(N) operations with N discretization points along Γ; (vi) The small-scale decomposition (SSD) [4] removes a severe restriction on the size of a time-step that arises from the Young–Laplace boundary condition.

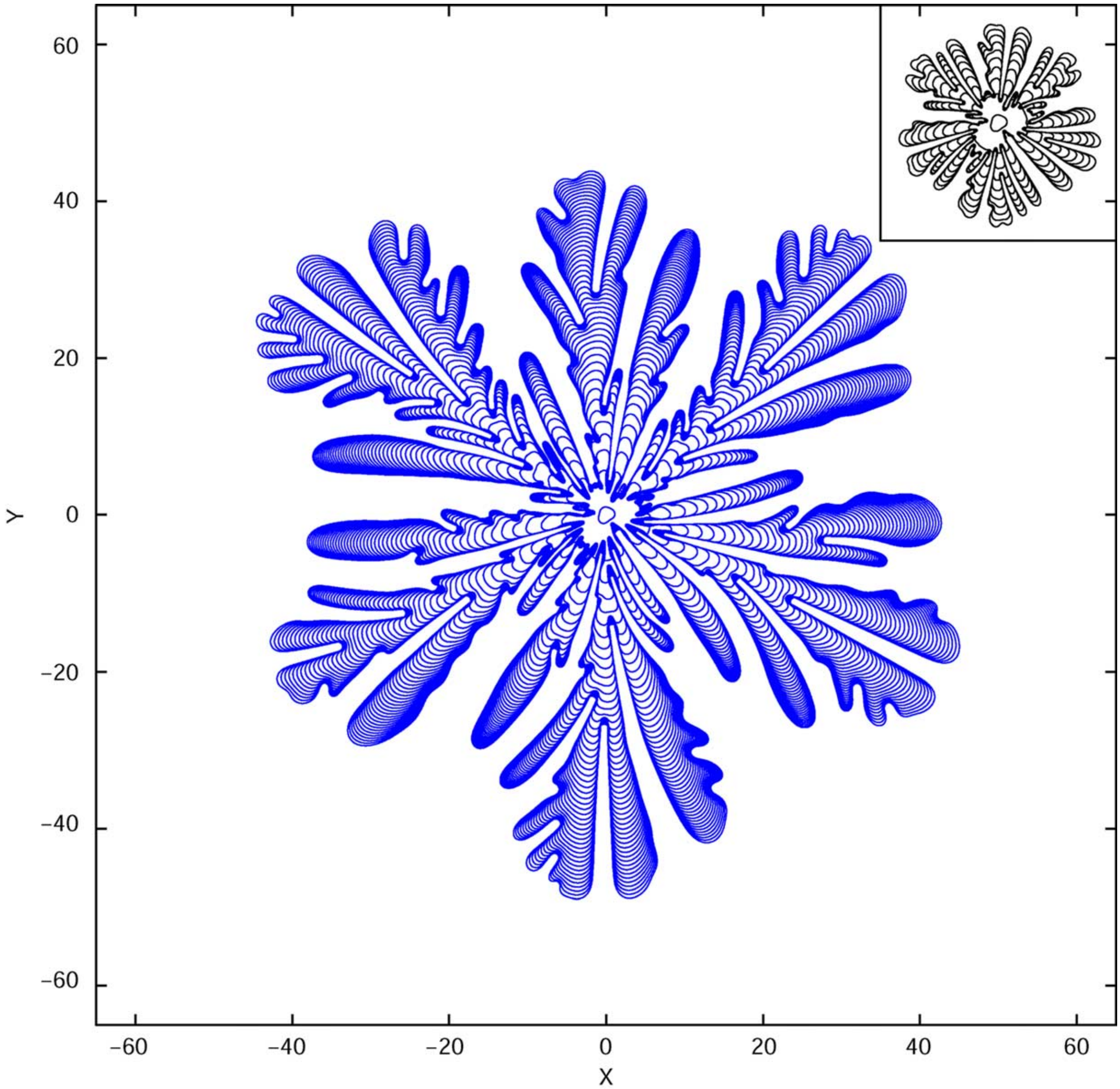

Fig. 1. The world's largest viscous fingering simulation at $T = 500$. The computation completes in 50 days on a modern desktop PC. The resolution has been increased as necessary through $N = 4096$, $N = 8192$, $N = 16{,}384$, $N = 32{,}768$ to resolve the expanding interface during the 500,000 time steps taken during the simulation. Inset: The reference viscous fingering computation published in 1994 by Hou et al. [4]. The computation required 50 days in 1994 to reach $T = 45$ in 45,000 timesteps using at most $N = 8192$ grid points along the interface (shown to scale).