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A new time–space domain high-order finite-difference method for the acoustic wave equation

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ABSTRACT

A new unified methodology was proposed in Finkelstein and Kastner (2007) [39] to derive spatial finite-difference (FD) coefficients in the joint time–space domain to reduce numerical dispersion. The key idea of this method is that the dispersion relation is completely satisfied at several designated frequencies. We develop this new time–space domain FD method further for 1D, 2D and 3D acoustic wave modeling using a plane wave theory and the Taylor series expansion. New spatial FD coefficients are frequency independent though they lead to a frequency dependent numerical solution. We prove that the modeling accuracy is 2nd-order when the conventional $(2M)$th-order space domain FD and the 2nd-order time domain FD stencils are directly used to solve the acoustic wave equation. However, under the same discretization, the new 1D method can reach $(2M)$th-order accuracy and is always stable. The 2D method can reach $(2M)$th-order accuracy along eight directions and has better stability. Similarly, the 3D method can reach $(2M)$th-order accuracy along 48 directions and also has better stability than the conventional FD method. The advantages of the new method are also demonstrated by the results of dispersion analysis and numerical modeling of acoustic wave equation for homogeneous and inhomogeneous acoustic models. In addition, we study the influence of the FD stencil length on numerical modeling for 1D inhomogeneous media, and derive an optimal FD stencil length required to balance the accuracy and efficiency of modeling. A new time–space domain high-order staggered-grid FD method for the 1D acoustic wave equation with variable densities is also developed, which has similar advantages demonstrated by dispersion analysis, stability analysis and modeling experiments. The methodology presented in this paper can be easily extended to solve similar partial difference equations arising in other fields of science and engineering.

1. Introduction

Finite-difference (FD) methods have been widely used in seismic modeling (e.g. [1–4]) and seismic migration (e.g. [5–8]) since these methods are fairly easy to implement. They also require relatively small memory and computation time compared to other purely numerical methods such as finite elements (e.g. [9]) and modified direct solution methods (DSM) [49,50]. A 2nd-order FD scheme is usually used for approximating temporal derivatives to perform wave field recursion

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effectively and stably; this, however, limits the accuracy of modeling. A smaller time step or grid size may increase the modeling precision but will require more computation time. Many methods, such as high-order, staggered-grid and implicit methods, have been developed to improve the accuracy which do not markedly increase the computation cost.

High-order FD on the space derivatives is a popular method to increase modeling accuracy (e.g. [10–19]). A low-order FD algorithm uses a shorter operator but needs more grid points for discretization. A high-order FD algorithm uses a longer operator but needs fewer grid points. It has been demonstrated that high-order FD schemes have practical advantages when applied to the scalar wave equation [10]. The FD schemes with accuracy of any or all orders have been derived for the first-order derivatives and used to solve wave equations (e.g. [10,11]). FD coefficients are generally determined by a Taylor series expansion (e.g. [10,43]) or by optimization (e.g. [11,44–46,48,51,52]). The effect of reducing the formal order of accuracy of a FD scheme in order to optimize its high-wavenumber performance was investigated [47] using the 1D nonlinear unsteady inviscid Burgers' equation and it was found that the benefits of optimization do carry over into nonlinear applications. FD operators, accounting explicitly for the amplitude spectrum, give more accurate results than Taylor or optimum operators in the elastic wave equation modeling [45]. The modified FD operators were derived by optimally minimizing the numerical dispersion of P- and S-velocities as an indirect consequence of their minimizing the error of synthetic seismograms, and the accuracy of synthetic seismograms computed using the modified operators was greatly improved as compared to conventional FD operators [51]. Using the Taylor expansion, the FD method with any even-order accuracy has been developed for arbitrary-order derivatives [14] and utilized to simulate wave propagation in two-phase anisotropic media [19].

Compared with conventional-grid FD methods, staggered-grid FD methods have greater precision and better stability and have been widely used in seismic modeling. An initial work for modeling of elastic wave equations was reported in [20]. A second-order velocity-stress staggered FD schemes for modeling SH-wave and P-SV wave propagation in generally heterogeneous media was proposed [21,22]. This method was used to simulate elastic wave propagation in 3D media (e.g. [23–28]). When the medium possesses discontinuities with large contrasts, modeling of elastic waves with an explicit FD scheme on a staggered grid causes instability problems. Rotated staggered grids, where all the medium parameters are defined at appropriate positions within an elementary cell for the essential operations, have also been used in the FD method [29–32].

Implicit FD methods have also been developed to improve the modeling accuracy. There are two kinds of implicit method. One is the implicit FD on temporal derivatives for the elastic wave equations [33]. It expresses a temporal derivative value at some point at a future time in terms of the values of the variable at that point and at its neighboring points at present time, past times, and future times. The implicit method for temporal derivatives has been used successfully in seismic migration algorithms [34]. The other implicit method is the implicit FD on spatial derivatives (e.g. [41,42]). This method expresses the spatial derivative value at some point in terms of the function values at that point and at its neighboring points and the derivative values at its neighboring points. A compact FD method [2] is such an implicit method, which has been widely applied in modeling (e.g. [35–38]).

Since the explicit high-order FD on temporal derivatives is usually unstable in the wave equation modeling (e.g. [17]), spatial derivatives are used to replace high-order temporal derivatives (e.g. [10,17]) to increase the accuracy of temporal derivatives with additional computational cost. Our goal is to derive new FD coefficients for spatial derivatives that can increase the accuracy of acoustic wave modeling without increasing calculation amount. Generally, most FD methods determine the FD stencils for spatial derivatives only in the space domain. However, the seismic wave propagation calculation is done both in space and time domains. If these stencils are directly used to solve the wave equations, the dispersion will always exist and may be very large. To address this issue, a unified methodology in [39] has been proposed to derive the FD coefficients in the joint time–space domain. The key idea of this method is that the dispersion relation is completely satisfied at designated frequencies; thus several equations are formed and the FD coefficients are obtained by solving these equations. Thus one can obtain dispersion free simulation at a given frequency. However, since different frequencies require different dispersion criterion, this method may not be very useful in practical applications which require time domain simulation where the source spectrum contains a continuous band of frequencies. This FD method was developed further for the 1D lossless and boundless wave equation in [40] and its spatial FD coefficients were determined at one designated frequency to obtain arbitrary-order accuracy.

In this paper, we develop a unified methodology similar to the method [39,40] and employ the Taylor series expansion of dispersion relation to derive the FD coefficients in the joint time–space domain. We prove that the modeling accuracy is of 2nd-order when the conventional $(2M)$th-order space domain FD and the 2nd-order time domain FD stencils are directly used to solve the acoustic wave equation. The new spatial FD coefficients for the 1D acoustic wave equation modeling are determined by the Courant number and the space point number and are independent of frequency. We demonstrate that while spatial difference coefficients of 1D in [39] depend on the space point number, the Courant number and frequencies, our difference coefficients for space derivatives are determined by the space point number and the Courant number only. For the 1D modeling, the accuracy can be improved from 2nd-order of the conventional method to $(2M)$th-order of the new method when $2M+1$ points are involved in the spatial derivatives and 3 points in the temporal derivatives. This conclusion can be obtained from [40]. Equations for solving 1D spatial FD coefficients derived in our paper are equivalent to those in [40]. However, we derive their explicit expressions for the first time. For 2D modeling, spatial difference coefficients in [39] depend on the space point number, the Courant number, frequencies and angles. Our difference coefficients for space derivatives in 2D are determined by the space point number, the Courant number and the angle. Moreover, we find an optimal angle $\pi/8$ which enables the modeling to reach $(2M)$th-order accuracy along eight directions; therefore, our difference coefficients are independent of angle. For the new 3D method, $(2M)$th-order accuracy can be reached along 48 directions