Journal of Computational Physics 228 (2009) 7986–8014

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Coupling Biot and Navier–Stokes equations for modelling fluid–poroelastic media interaction

Santiago Badia [a,1], Annalisa Quaini [b,*,2], Alfio Quarteroni [b,c,3]

[a] International Center for Numerical Methods in Engineering (CIMNE), Universitat Politècnica de Catalunya, Jordi Girona 1-3, Edifici C1, 08034 Barcelona, Spain
[b] Chair of Modelling and Scientific Computing (CMCS), Institute of Analysis and Scientific Computing (IACS), École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
[c] MOX – Modellistica e Calcolo Scientifico Dipartimento di Matematica "F. Brioschi", Politecnico di Milano via Bonardi 9, 20133 Milano, Italy

ARTICLE INFO

Article history:
Received 2 March 2009
Received in revised form 29 June 2009
Accepted 21 July 2009
Available online 30 July 2009

MSC:
65M12
65M60

Keywords:
Darcy's problem
Biot system
Poromechanics
Fluid–structure interaction
Stabilized finite elements
Hemodynamics

ABSTRACT

The interaction between a fluid and a poroelastic structure is a complex problem that couples the Navier–Stokes equations with the Biot system. The finite element approximation of this problem is involved due to the fact that both subproblems are indefinite. In this work, we first design residual-based stabilization techniques for the Biot system, motivated by the variational multiscale approach. Then, we state the monolithic Navier–Stokes/Biot system with the appropriate transmission conditions at the interface. For the solution of the coupled system, we adopt both monolithic solvers and heterogeneous domain decomposition strategies. Different domain decomposition methods are considered and their convergence is analyzed for a simplified problem. We compare the efficiency of all the methods on a test problem that exhibits a large added-mass effect, as it happens in hemodynamics applications.

1. Introduction

The interaction between a free fluid and a deformable porous medium is found in a wide range of applications: groundsurface water flow, geomechanics, reservoir engineering, filters design, seabed–wave or blood-vessel interactions. Let us focus on the latest application. From the arterial lumen (where blood flows), the blood enters the artery walls. Hence, in simulating the blood–artery interaction *neglecting the porosity of the artery wall means to disregard an important feature*. Modeling the fluid–poroelastic interaction in an accurate and efficient way represents a step forward towards the numerical simulations of complex clinical problems. For instance, it permits to simulate how low-density lipoproteins (LDL) or drugs are filtrated into the tissue.

* Corresponding author.
E-mail addresses: sbadia@cimne.upc.edu (S. Badia), annalisa.quaini@gmail.com (A. Quaini), alfio.quarteroni@epfl.ch (A. Quarteroni).
[1] Supported by the European Community through the Marie Curie contract *NanoSim* (MOIF-CT-2006-039522).
[2] Supported by the Swiss National Science Foundation under grant 200021-109378/1.
[3] Support of the European Research Council Advanced Grant "Mathcard, Mathematical Modelling and Simulation of the Cardiovascular System", Project ERC-2008-AdG 227058.

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2009.07.019

The classical fluid–structure interaction problem that appears in hemodynamics (Navier–Stokes coupled to the elasticity for thin structures) has been broadly studied (see, e.g., [61,20] and references therein). Many works have been devoted also to the Navier–Stokes/Darcy coupling (see, e.g., [58,59,3] and references therein) to simulate mass transport from the arterial lumen to the arterial walls and inside the walls, when the latter are supposed to be undeformable. The fluid–poroelastic structure interaction (FPSI) problem couples the Navier–Stokes equations for an incompressible fluid to the Biot problem, the latter governing the motion of a saturated poroelastic medium. FPSI has received much less attention. For hemodynamics applications, the most salient work is [49], where the Biot system is stated in terms of the structural velocity $\boldsymbol{u}_s$ (or displacement), filtration flux $\boldsymbol{q}$, and pressure p_p. The coupled system is linearized by Newton's method and solved by a monolithic solver. A simplified FPSI system appearing in hemodynamics has also been considered in [19]. Therein, the Biot system is written in terms of $(\boldsymbol{u}_s, p_p)$ only, after neglecting the inertia terms in Darcy's law. The fact that $\boldsymbol{q}$ does not appear in the formulation requires to introduce artificial boundary conditions on the interface between the lumen and the poroelastic vessel medium.

Even though it is common practice to write the Darcy problem as a pressure Poisson equation, we will not adopt this approach here for several reasons. The original Darcy's law is a transient problem (see [29]), and inertia terms must be neglected in order to obtain the pressure Poisson problem. Much more critical is the fact that the Poisson problem fails to approximate non-smooth pressures in areas with jumps of physical parameters (e.g., hydraulic conductivity or porosity). The local pressure instabilities appearing in these areas are well-known in soil consolidation computations and motivated mixed formulations in [71]. However, the main reason why the Darcy's system has to be stated in mixed form is that we want to couple this problem with the Navier–Stokes equations via proper transmission conditions. The fact that $\boldsymbol{q}$ appears explicitly in the formulation is of great importance, because it allows to enforce the proper boundary conditions at the fluid–porous structure interface (see Section 4.1).

The numerical approximation of FPSI problems is challenging due to the three inf–sup conditions that need to be fulfilled in order for the coupled problem to be well-posed: the inf–sup condition for the fluid sub-problem and the inf–sup conditions for both incompressible elasticity and Darcy's problem for the poroelastic subproblem. While there exists a great variety of stabilization techniques for the incompressible Navier–Stokes equations (e.g., [17,65]), very few works deal with the stabilization of the Biot system in mixed form (no pressure Poisson equation is used). For instance, the Biot system in terms of $(\boldsymbol{u}_s, \boldsymbol{q}, p_p)$ has been approximated using a characteristic-based splitting algorithm in [71] and using penalty terms in [21]. In this work, we introduce a residual-based stabilization technique motivated by the variational multiscale method (VMS). This technique, introduced in [43], allows to use finite element spaces that do not satisfy the inf–sup conditions at the discrete level. In fact, the associated algebraic system is quite involved, and the use of the same finite element spaces for all the velocities and pressures greatly simplifies the discretization and the enforcement of transmission conditions. We will consider linear Lagrangian elements for all the unknowns in the numerical experiments.

We extend to FPSI problems some of the strategies adopted for fluid–elastic structure interactions. Unlike [49,19], we choose a fixed point method for the linearization of the Navier–Stokes/Biot coupled system. In this way, it is easy to consider the semi-implicit versions of all the algorithms, i.e. only one fixed point iteration is performed per time step. Semi-implicit methods enable us to better understand the Navier–Stokes/Biot coupling since nonlinearities are explicitly treated. To solve the linear FPSI system, we propose to extend both the monolithic approach introduced in [7] and partitioned procedures based on domain decomposition preconditioners. At the best of our knowledge, it is the first time that a modular approach is adopted for FPSI problems. A fluid–structure algorithm is said to be modular when it only requires interface data transfer between the two codes, without any modification of the sources. A modular algorithm allows to reuse existing (and already optimized) fluid and structure codes. Among all the partitioned procedures, we focus our attention on the Dirichlet–Neumann, Robin–Neumann, and Robin–Robin algorithms (see, e.g., [66]).

In summary, the main novelty of this work consists in: the development of a residual-based stabilized finite element method for the Biot system; the use of a semi-implicit monolithic method for the Navier–Stokes/Biot system; the extension of domain decomposition techniques to the FPSI problem and the comparison with non-modular solvers.

In Section 2 we state the Navier–Stokes/Biot coupled problem in its differential form, specifying the coupling conditions which lead to a mathematically well-posed problem. The variational formulation of the coupled problem is tackled in Section 3. In Section 4 we develop a $(\boldsymbol{u}_s, \boldsymbol{q}, p_p)$ residual-based stabilized formulation of the Biot system. The matrix form of the Navier–Stokes/Biot system associated to the fully discretized and linearized problem is described in Section 5. Sections 6 and 7 present our monolithic approach and the partitioned procedures we apply to solve the linear system. Finally, in Section 8, we carry out some numerical experiments on simplified $2d$ problems representing blood-vessel systems.

2. Problem setting

Suppose that a bounded, polyhedral, and deformable domain $\Omega_t \subset \mathbb{R}^d$ ($d = 2, 3$, being the space dimension, and $t \in [0, T]$ the time) is made up of two regions, Ω_t^f and Ω_t^p, separated by a common interface $\Sigma_t = \partial\Omega_t^f \cap \partial\Omega_t^p$. The first region Ω_t^f is occupied by an incompressible and Newtonian fluid, and the second one Ω_t^p is occupied by a fully-saturated poroelastic matrix. Both domains depend on time. Here, we denote by $\boldsymbol{n}$ the unit normal vector on the boundary $\partial\Omega_t^f$, directed outwards into Ω_t^p, and by $\boldsymbol{t}$ the unit tangential vector orthogonal to $\boldsymbol{n}$. We assume the boundary $\partial\Omega_t$ (and so $\boldsymbol{n}$ and $\boldsymbol{t}$) to be regular enough.