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Finite difference time domain dispersion reduction schemes

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Abstract

The finite-difference-time-domain (FDTD), although recognized as a flexible, robust and simple to implement method for solving complex electromagnetic problems, is subject to numerical dispersion errors. In addition to the traditional ways for reducing dispersion, i.e., increasing sampling rate and using higher order degrees of accuracy, a number of schemes have been proposed recently. In this work, an unified methodology for deriving new difference schemes is presented. It is based on certain modifications of the characteristic equation that accompanies any given discretized version of the wave equation. The method is duly compared with existing schemes and verified numerically.

Keywords: Numerical methods; Finite difference time domain; Numerical dispersion error reduction; Wave equation; Electromagnetics

1. Introduction

The finite-difference-time-domain (FDTD) [1] has long been established as a flexible and robust method that is simple to implement for solving the complex electromagnetic problems. The method has thus become very popular, in particular in its second-order accurate, central differences variant known as the $(2,2)$ scheme. In spite of its advantages, the method is subject to numerical dispersion errors that tend to hinder its application in problems involving narrow pulses and large time spans.

Two traditional ways have been used for the purpose of reducing dispersion, i.e., increasing sampling rate and using higher order degrees of accuracy beyond the classic $(2,2)$ central differencing as used originally by Yee [1]. Higher order explicit approximations for Maxwell's equations, based on Yee's staggered grid, were introduced in [2]. The superior accuracy and efficiency over Yee's scheme was confirmed and quantified in [3,4]. Later, Liu showed [5] that the spatial part of the central difference second-order discrete wave equation can be arrived at either by using Yee's staggered uncolocated scheme or by his newly developed unstaggered scheme, where he used a backward difference on $\nabla \times \mathbf{H}$ and forward difference on $\nabla \times \mathbf{E}$. He then generalized his scheme to a higher order combination of upwind and the corresponding antisymmetric differencing. Further, he derived a fourth-order spatial discretization scheme as a special case with two options of time

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discretization, i.e., second-order leapfrog and fourth-order Runge–Kutta. Fang's (2,4) algorithm [2] was reformulated in [6] in a compact and simple way, that allowed for the investigation of its dispersion properties and interaction with different boundary conditions. The question of boundary condition for higher order schemes was also addressed in [7]. Other noteworthy contributions on explicit high order FDTD methods are [8], where a variant of Fang's (4,4) scheme was developed based on a variation on the fourth-order time discretization. Implicit compact high order methods for solving Maxwell's equations can be found in [9–13]. Alternative methods for accuracy enhancement may be categorized in belonging to one of three groups. The first and most widely used group has its roots in the early work by Miranker [14]. Realizing that the truncated Taylor series may not be the most accurate way to discretize the wave equation, he suggested a minimization of a functional containing Fourier components of the solution. Miranker produced explicit time marching schemes by analytically solving constrained minimization problems with quadratic cost. Due to the complexity of the analytic calculation required, Miranker's method went relatively unnoticed until rediscovered in the early 90's by Lele [15], who constructed a class of highly accurate compact schemes for first, second and higher derivatives. Lele attained high accuracy by using a 7-point stencil, although without exploiting its full capacity in order to get high order Taylor series accuracy; rather, he used a portion of the series to make the discrete operator equal to the continuous one at three predefined high frequencies. Tam and Webb [16] devised an explicit high order spatial and temporal algorithm for solving the linearized Euler equation encountered in acoustics. They referred their scheme as a Dispersive Relation Preserving one (DRP). Haras and Ta'asan [17] used Miranker's minimization approach combined with Lele's implicit compact discretization to create their version of a high order scheme. Liu [5] used a discretized first derivative operator (upwind) and its antisymmetric one to create a nonstaggered scheme to solve Maxwell's equation. The coefficients in this scheme are determined by solving a Miranker-type functional numerically. Recently, Wang and Teixeira published a series of papers [18–21] employing artificial dispersion and adjustable coefficients with high frequency digital filtering to create optimized DRP schemes. Zygiridis and Tsiboukis [22] and Sun and Trueman [23] devised an optimized scheme based on a staggered (2,4) stencil. Another recent work in this context is [24].

The second group is based on Mickens's nonstandard FDTD (NSFDTD) method [25]. In this method, one creates difference schemes with solutions that are exact replicas of the solutions to their continuous PDE counterparts, using a generalization of the traditional approximation to the derivative operator. As opposed to the complex minimization processes required to construct the difference schemes of the first group, Mickens's scheme finds a suitable function to generalize the derivatives of a specific PDE thus expressing the difference scheme coefficients in an explicit manner. This method was applied to CEM in [26], where the wave equation was dicretized by using a linear combination of conventional in addition to diagonal discretizing (2,2) schemes in order to improve isotropy properties at a single predefined frequency. This scheme was later modified [27] to accommodate Yee's staggered grid, and to improve stability and provide time step analysis [28]. In [29], Mickens's strategy was utilized to better discretize 1D, 2D and 3D Helmholtz equations and the temporal Maxwell's equations. Recently, Yang and Balanis have devised in [30,31] an improved nonstandard finite difference to mitigate anisotropy in (2,2) and (2,4), respectively, Yee type stencils.

Finally, the third group includes [32] where Maxwell's equations are used in their integral form to produce a modified (2,4) explicit algorithm with two adjustable parameters to minimize phase error. In [33,34], Yee's and Bi's [35] types of grids, having complementary phase behavior are combined to produce an almost isotropic scheme. Finally, a moment-like expansion of the fields has been employed to produce more accurate solutions. The use of scaling and wavelet functions in approximations for the spatial derivative operator was shown in [36], and a recent flexible local approximation method (FLAME) [37] makes use of functions that are solutions to Maxwell's equations as a basis to expand the field components in both time and space.

In this work, we develop an unified methodology for deriving new difference schemes that can accommodate arbitrary requirements for reduced phase or group velocity dispersion errors, defined over any region in the frequency-direction space. The preferred starting point for this development is an unified formulation for all schemes involving the characteristic equation, as outlined in Section 2. A general strategy for developing various dispersion reduction schemes is based on certain systematic modifications of the characteristic equations, as presented in Section 3. This strategy is then spelled out for cases such as several 1D wave equation FDTD schemes (Section 4) and the 2D (2,2) FDTD scheme (Section 5). Analytical comparison with other schemes is done in Section 6. Analysis of dispersion curves is carried out in Section 7, where regions of better