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2D nearly orthogonal mesh generation with controls on distortion function

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Abstract

A method to control the distortion function of the Ryskin and Leal (RL) orthogonal mesh generation system is presented. The proposed method considers the effects from not only the local orthogonal condition but also the local smoothness condition (the geometry and the mesh size) on the distortion function. The distortion function is determined by both the scale factors and the averaged scale factors of the constant mesh lines. Two adjustable parameters are used to control the local balance of the orthogonality and the smoothness. The proposed method is successfully applied to several benchmark examples and the natural river channels with complex geometries.

Keywords: Mesh generation; Distortion function; Scale factor

1. Introduction

It is generally accepted that regardless of the discretization method, the quality of a computational mesh, usually characterized by the orthogonality and the smoothness, has significant influences on the solutions of the non-linear partial differential equations (PDE). Although extensive research (for examples, see [1–14]) has been made on high quality mesh generation, the generation of orthogonal mapping with adequate smoothness in geometrically complex domains still remains a challenge.

Many methodologies and techniques have been proposed for orthogonal mesh generation since late 1970s (see [1–12,14]). Conformal mapping is the most well-known orthogonal mapping. It is simple, efficient and easy to use. However, because it requires equal scale factors in all directions, the conformal mapping may cause folded meshes at the concave boundaries. Ryskin and Leal [10] proposed a covariant Laplace equation

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system for orthogonal mapping (often referred as RL system). The RL system has been the objective of many researchers (see [1–6,8,11,14]). The focus of these researches was the determination of the distortion function *f*, which generally cannot be prescribed arbitrarily. Eça [5] classified three different treatments of the distortion function: (1) calculate *f* from its definition at the boundaries and then obtain the values in the domain by interpolation or by solving a Laplace equation (see [10,11]); (2) specify a class of admissible functions for *f* based on the quasi-conformal mapping theory to guarantee a unique solution (see [3,4]); and (3) calculate *f* from its definition for the whole domain (see [1,2,5]).

According to its definition, the distribution of the distortion function *f* actually describes the mesh density distribution. In the first two methods, the distortion function *f* is controlled algebraically or numerically, while in the third method there are no controls on the distortion function *f*. Therefore, the above three methods can be simply categorized into two groups: methods with controls on distortion function and methods without controls on distortion function. For methods without controls on the distortion function *f*, the mesh density or aspect ratio is controlled only by the local orthogonal condition and consequently its distribution in the whole domain is not predicable. It is already pointed out by Eça [5], Akcelik et al. [1] and Zhang et al. [14] that the RL system may cause serious mesh distortions or overlapping in complex geometries when using the "weak constraint" method with the specified boundary point distribution for all boundaries.

In this paper, a new method of formulating the distortion function is proposed. In addition to the local scale factors, the globally averaged scale factors of the constant mesh lines are also used to evaluate the distortion function *f*. Local balance of the orthogonality and the smoothness is controlled by two adjustable empirical parameters which are evaluated automatically based on the deviation from the local smoothness condition. Several examples and applications are used to test and illustrate the proposed method. It is demonstrated that this method is effective and easy to use.

2. Elliptic mesh generation systems

The RL system proposed by Ryskin and Leal [10] and the conformal mapping system are two classical elliptic orthogonal mapping systems. The former, a covariant Laplace equation system, can be easily derived in a way analogue to the Laplace equations for stream function and velocity potential function. In the RL system, the orthogonal mapping between the physical coordinates ($x^i(\equiv x, y)$, $i = 1, 2$) and the computational coordinates ($\xi^i(\equiv \xi, \eta)$, $i = 1, 2$) can be described using the following covariant equations:

$$\frac{\partial}{\partial \xi}\left(f\frac{\partial x}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{1}{f}\frac{\partial x}{\partial \eta}\right) = 0 \tag{1a}$$

$$\frac{\partial}{\partial \xi}\left(f\frac{\partial y}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{1}{f}\frac{\partial y}{\partial \eta}\right) = 0 \tag{1b}$$

where the distortion factor *f* (also called aspect ratio) is defined as the ratio of the scale factors in ξ and η directions:

$$f = \frac{h_\eta}{h_\xi} = \left(\frac{x_\eta^2 + y_\eta^2}{x_\xi^2 + y_\xi^2}\right)^{1/2} \tag{2a}$$

$$h_\xi = g_{11}^{1/2}, \quad h_\eta = g_{22}^{1/2} \tag{2b}$$

where $x_\xi = \partial x/\partial \xi$ and so forth.

In Eq. (2), the metric tensor g_{ij} represents the physical features of a computational mesh and it is defined as follows:

$$g = \begin{vmatrix} (x_\xi^2 + y_\xi^2) & (x_\xi x_\eta + y_\xi y_\eta) \\ (x_\xi x_\eta + y_\xi y_\eta) & (x_\eta^2 + y_\eta^2) \end{vmatrix} \tag{3}$$