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A consistent dynamic localization model for large eddy simulation of turbulent flows based on a variational formulation

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Abstract

In this study, a new approach for the dynamic localization model, which was originally proposed in Ghosal et al. [S. Ghosal, T.S. Lund, P. Moin, K. Akselvoll, A dynamic localization model for large-eddy simulation of turbulent flows, J. Fluid Mech. 286 (1995) 229–255], is described. This approach is integrated in a consistent manner into large eddy simulation based on a variational formulation. As a result, the variationally formulated condition for the model parameter is considered as an additional equation in a resulting system of two variational equations. This variational system may then be implemented using either a finite element or a finite volume method. The new version of the dynamic localization model proposed in this work has three advantages compared to the original dynamic localization model in Ghosal et al.: it relies on a simpler formulation overall, it obviates any iterative solution procedure, and it requires the solution of a number of small independent local equations instead of one large global equation. These three advantages make its solution theoretically easier and computationally more efficient. The new consistent dynamic localization model is tested for two different numerical flow examples, turbulent flow in a channel and turbulent flow in a planar asymmetric diffuser.

Keywords: Turbulence; Large eddy simulation; Variational formulation; Subgrid-scale modeling; Dynamic localization model

1. Introduction

The modeling of the unresolved or subgrid scales is a crucial aspect of large eddy simulation (LES) of turbulent flows. In the traditional LES, two different ways of subgrid-scale modeling may generally be distinguished according to [27]. One approach intends to approximate the subgrid-scale stress tensor $\boldsymbol{\tau}$ itself. This strategy is called *structural modeling*. Another approach aims at modeling the (energetic) action of the unresolved (or subgrid) scales on the resolved scales rather than modeling the tensor itself. This second strategy is called *functional modeling*. A very popular form of functional modeling relies on the subgrid (or eddy) viscosity concept, which is based on the Boussinesq turbulent (or eddy) viscosity assumption. According to

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this concept, the deviatoric part of τ is approximated by a product of a subgrid viscosity ν_T and the rate-of-strain tensor of the resolved scales.

The remaining unknown in this form of functional modeling is the subgrid viscosity ν_T, and several approximations for ν_T have been proposed in the meantime. The Smagorinsky [29] model was the first proposal in this context and is still a commonly used one due to its simplicity. However, that model is still based on an unknown parameter C_S. Choosing a constant value for this parameter over the entire domain has been proven to be an inadequate approach in most of the cases, although recent integrations of this simple constant-coefficient-based Smagorinsky model into a multiscale environment have revitalized it and yielded very good results for a number of test cases (see, e.g., [7,9,13,14,17]). Within the multiscale environment, the subgrid-scale model is directly applied only to the smaller resolved scales after a separation of the range of resolved scales. The reader may, for instance, consult the initial publication on the variational multiscale method for LES (i.e., [12]) for the basic idea. For a specific finite-volume or combined finite-element/finite-volume method within the variational multiscale LES, it is referred to [7,17], respectively. A comprehensive overview has recently been provided in [8], see also references therein for further elaboration.

An important improvement of the Smagorinsky model was introduced in [5], where the model parameter was determined as a function of position and time by way of a dynamic algorithm. It should be remarked that this dynamic procedure is not restricted to the Smagorinsky model as the underlying model, although it has mostly been used with that model. The original formulation of the dynamic algorithm in [5] contained a mathematically inconsistent assumption, which disregarded the fact that C_S is a rapidly varying function of position, as discussed in [22].

This inconsistency was later overcome in [6] by the introduction of the dynamic localization model. Furthermore, ad hoc schemes were addressed in this publication, which were usually applied in practical simulations to prevent them from becoming unstable. By using those ad hoc schemes, the application of the dynamic model in several problem configurations was enabled, without, however, being justified except in a heuristic way. Overall, the study in [6] aimed at "putting the dynamic modeling procedure on firm theoretical foundations, so that the method could be applied to arbitrary inhomogeneous flows without recourse to ad hoc procedures". After all, however, the actual way of calculating the model parameter based on a variationally formulated condition for C_S, which eventually had to make use of Fredholm's integral equation of the second kind for its solution, appeared to be rather complicated. Moreover, an iterative procedure was necessary to actually solve this integral equation in practical calculations, which made it a computationally expensive part of the overall simulation. This study was complemented by another study in [2], which addressed the issue of representing backscatter in the dynamic localization model by a stochastic modeling approach.

In an approximate version of the dynamic localization model proposed in [26], an iterative procedure was avoided by using an approximation in time, which, however, was accurate only up to the respective order of accuracy of the temporal approximation scheme. Moreover, this method faced, on the one hand, potential numerical instabilities and, on the other hand, potential inaccuracies in the case of rapid variations in the temporal evolution of the function, as admitted in [26]. However, the authors assumed the evolution of C_S to be a fairly slowly varying function of time due to the temporal filtering, which is implicitly introduced by the spatial filtering. Another recent approach in [31] makes the dynamic localization model less demanding by relying on an averaging procedure over homogeneous coordinate directions. However, this approach is not applicable to arbitrary inhomogeneous flows.

In the present study, LES is based on a variational formulation. In contrast to a traditional filter-based formulation, the resolution of the underlying numerical discretization is used to define the resolved part of the velocity $\boldsymbol{u}^h$, with the superscript h indicating the characteristic length scale of the discretization. It should be remarked that this is actually a usual way of defining the resolved scales in practical LES, whenever the respective discretization is assumed to act as an implicit filter, and no further explicit filter is applied. The reason for introducing a subgrid-scale model in the variational formulation is mathematically different from the usual necessity of introducing a model term due to the appearance of a subgrid-scale stress tensor in the strong formulation of the Navier–Stokes equations in a traditional LES. Nevertheless, the physical necessity of accounting for the missing effect of unresolved scales on the resolved scales is the same in both cases. In order to account for this effect in the present study, it is resorted to the subgrid viscosity concept and, furthermore, the Smagorinsky model as one way of functional modeling, as mentioned above. Within a variational formu-