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A new category of Hermitian upwind schemes for computational acoustics – II. Two-dimensional aeroacoustics

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Abstract

This is the second paper of a series in which we build and study third-order Hermitian upwind schemes for numerically solving linear aeroacoustics. In this paper, we extend the method to non-stationary flows. For this purpose, we employ a family of six-wave models to discretize the space operator. This family is parametrized by an "acoustic propagation angle". From this wave modelling, specific boundary conditions are proposed to treat effectively subsonic/supersonic boundary conditions. A sequence of numerical simulations is then carried out and makes it possible to examine the effectiveness of the scheme.

Keywords: Hermitian schemes; Upwind discretization; Wave-modelling; Aeroacoustics; Time-fluctuation; Outflow/radiation boundary conditions

1. Introduction

This is the second article devoted to the construction of a third-order Hermitian method. This is a method devised to numerically solve the initial boundary value problem associated with the linear acoustics [1]. Hermitian interpolation, wave decomposition and non-reflecting boundary conditions are combined to produce a third-order compact scheme: the "Δ-P3 scheme". In this paper, we continue our work and extend the method to the case of linear aeroacoustics.

In a uniform mean-flow, the linear aeroacoustic equations support three types of waves, namely, the acoustic, the entropy and the shear (or vorticity) waves. The acoustic waves are isotropic, non-dispersive, non-dissipative and propagate with the speed of sound. The entropy and shear waves are non-dispersive, non-dissipative and directional; they are convected in the direction of the mean-flow with the same speed as the flow. There is no guarantee in most of the standard computational aeroacoustic (CAA) schemes that the true physics of the multi-dimensional problem is modelled correctly, since the waves can be propagated only in the directions defined by the grid. Usually, this deficiency causes an inconsistency in the decomposition

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when the dominant characteristic direction present in the flow field lies oblique to the grid. The main objective of this work is to present a new category of schemes, which try to make the numerics as harmonious as possible with the physics.

The first important ingredient that defines our algorithm is the "decomposition step", defined in [1]. This step is underlined in this article. The decomposition can be regarded as a way of explaining with what the temporal derivative is due. This method, at first proposed by Roe [15] decomposes time-derivatives into a defined set of simple waves, all free of the grid orientation. Then, this decomposition is utilized to evolve in time the solution in a discrete cell by sending the disturbance caused by each individual wave to the point towards which it is propagated. To model the principal characteristics of linear aeroacoustics, we define a family of six-wave models parametrized by the acoustic propagation angle, θ. Thus, we can construct two wave-models based on physical considerations: the "wave-model A" ($\theta \equiv \pi/4$) and the "wave-model B" (θ is given by the local mean flow direction). By defining arbitrary propagation directions, some dissipation can be possibly added, but as demonstrated in [1], the two pairs of mutually orthogonal acoustic waves contribute to reduce this damping term. Furthermore, the resulting scheme remains linear, preventing specific numerical problems due to possible non-linear instabilities.

The second significant characteristic of our algorithm lies in the treatment of physical boundary conditions. For the linear aeroacoustic equations with a uniform mean flow, the acoustic waves are the principal outgoing waves except along the outflow boundary. Along the outflow boundary, the outgoing disturbances consist of a combination of acoustic, entropy and shear waves. Hence, special outflow boundary conditions are necessary to allow the exit of these disturbances from the computational domain, without any artificial reflection. For the remaining boundaries, the imposition of an acoustic radiation boundary condition remains sufficient. In [1], we examined radiation boundary conditions for the acoustics. This work extends these conditions to the aeroacoustics, in order to treat outflow boundary conditions in subsonic or supersonic regimes. The derived boundary conditions do not need the use of numerical techniques such as absorbing boundary conditions, artificial damping layer or mesh stretching (see [5,7] for more details on those techniques).

To test the effectiveness of wave modelling and the radiation and/or outflow boundary conditions proposed in this paper, a sequence of numerical experiments and comparisons with exact solutions of the linear aeroacoustic equations is carried out. In order to simplify the problem, the flow is supposed to be isentropic so that there is no need to model the entropy wave. The first numerical experiment relates to an acoustic wave pulse, initiated at the centre of the computation domain and superimposed on a uniform flow. The second numerical experiment models the generation and radiation of acoustic waves from a 2D shear layer. The unstable behaviour of a shear layer is excited by a fixed acoustic source. For this problem, radiation and outflow boundary conditions are simultaneously considered.

We are now ready to give a detailed description of the contents of this paper. In Section 2, we construct a third-order upwind scheme for the linear aeroacoustics. First, the decomposition stage is detailed. Thus, we develop two wave models to decompose the time-derivatives. Then, we briefly review the reconstruction stage, in order to generate a third-order Hermitian scheme, namely the "Δ-P3 scheme". Next, we derive the specific non-reflecting outflow/radiation boundary conditions for aeroacoustics. In Section 3, we present numerical tests. The scheme so generated is then compared with a "state of the art" numerical method designed for an efficient treatment of CAA problems. Finally, Section 4 makes a synthesis of this work and presents a conclusion.

2. High-order upwind schemes for acoustics in non-stationary flows

2.1. Governing equations

If there is a background isentropic flow in acoustic field, it transports both the acoustic and vorticity waves. Though we consider here the case where the background flow is uniform in the whole acoustic field, it introduces some additional parameters (flow Mach number and its direction) and makes it more difficult to develop an accurate and stable scheme such as that derived in [1].

In two dimensions, with the assumption that the mean flow remains isentropic, the governing equations for the three fluctuation variables, the pressure p and the two components of Cartesian velocities, u, v are written as follows: