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Localized artificial diffusivity scheme for discontinuity capturing on curvilinear meshes

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ABSTRACT

A simple and efficient localized artificial diffusivity scheme is developed for the purpose of capturing discontinuities on curvilinear and anisotropic meshes using a high-order compact differencing scheme. The artificial diffusivity is dynamically localized in space to capture different types of discontinuities such as a shock wave, contact surface or material discontinuity. The method is intended for use with large-eddy simulation of compressible transitional and turbulent flows. The method captures the discontinuities on curvilinear and anisotropic meshes with minimum impact on the smooth flow regions. The amplitude of wiggles near a discontinuity and the number of grid points used to capture the discontinuity do not depend on the mesh size. The comparisons between the proposed method and high-order shock-capturing schemes illustrate the advantage of the method for the simulation of flows involving shocks, turbulence and their interactions. The multi-dimensional formulation is tested on a variety of 1D and 2D, steady and unsteady, different types of discontinuity-related problems on curvilinear and anisotropic meshes. A simplification of the method which reduces the computational cost does not show any major detrimental effect on the discontinuity capturing under the conditions examined.

1. Introduction

Due to advances in computational power and numerical algorithms, the application of large-eddy simulation (LES) to transitional and turbulent compressible flows is the focus of significant recent research. The engineering motivation for compressible LES is to provide a more realistic turbulent flowfields than Reynolds-averaged Navier–Stokes simulations and to educidate the unsteady phenomena such as mixing, combustion, heat-transfer, sound-generation and unsteady loads which may be of interest.

Because of their spectral-like resolution, high-order compact differencing schemes [1] are an attractive choice for LES of transitional and turbulent flows to reduce dispersion, anisotropy and dissipation errors associated with the spatial discretization. High-order compact differencing schemes have been applied to practical applications [2–4], and they all have shown the capability of the algorithm. However, these central differencing schemes cannot be applied directly to flows that contain discontinuities. When flows contain steep gradients, such as shock waves, contact surfaces or material discontinuity, nonphysical spurious oscillations that make the simulation unstable are generated. The development of numerical algorithms

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that capture discontinuities and also resolve the scales of turbulence in compressible turbulent flows remains a significant challenge.

Several techniques to extend compact schemes to discontinuous flows have been proposed. Lee et al. [5] proposed a hybrid approach to capture discontinuities. In regions of strong shock waves, the compact differencing of convective fluxes is replaced locally by the essentially non-oscillatory (ENO) scheme [6]. Similarly, Rizzetta et al. [7] introduce hybridization of the compact differencing scheme with Roe's upwind-biased scheme [8]. Visbal and Gaitonde [9] developed an adaptive filter method in which the compact scheme is coupled with a locally reduced-order filter to capture discontinuities. Both approaches require a detector to identify the smooth and non-smooth regions in the flow. The choice of an effective discontinuity detector remains a bottleneck for these methods when applied to complex applications. Additionally, hybrid schemes, such as a hybrid of an accurate linear scheme and a robust non-linear shock-capturing scheme, can possibly cause numerical instabilities when multiple discontinuities are closely located (the grid points separating them are very few).

An attractive alternative to these methods has been proposed by Cook and Cabot [10,11], Fiorina and Lele [12] and Cook [13] by dynamically adding localized high-wavenumber biased artificial diffusivity where needed, to capture discontinuities using high-order compact differencing schemes. The main feature of the artificial diffusivity is to suppress the unresolved high frequency content of the flowfield on a given mesh to capture the discontinuity with minimal effects by smearing the discontinuity over a numerically resolvable scale. Also, in the limit of grid spacing $\Delta \to 0$, the artificial diffusivity vanishes and the governing equations converge to the original Navier–Stokes equations. Main advantages of the method are its simplicity, low computational cost, automatic deactivation in smooth regions (high-resolution characteristics of a high-order compact scheme is preserved in smooth regions), easy to implement in an existing code, design to provide high-wavenumber biased damping, and the lack of a discontinuity detector or weighting/hybrid scheme. All these advantages are desirable for compressible LES of the flows involving shock, contact, and material discontinuities, turbulence and their interactions. In the previous work the method was shown to work well on 1D and 2D shock-related problems. However, most of the test cases in the previous works used uniformly spaced Cartesian coordinate systems. Therefore, the extension of the method to curvilinear and anisotropic meshes is still an open issue. This extension is necessary for the method to be useful for practical applications.

The objective of this paper is to establish the methodology for capturing discontinuities in a curvilinear coordinate framework using a high-order compact scheme. Simple and efficient localized high-wavenumber biased artificial diffusivity scheme on curvilinear and anisotropic meshes is proposed. The original formulation is also simplified to reduce the computational costs while achieving better representation of high-order derivatives. The performance of the method will be assessed on a 2D smooth/non-smooth flows and several 1D and 2D, steady and unsteady, different types of discontinuity-related problems.

2. Mathematical models

2.1. Governing equations

The compressible Navier–Stokes equations for an ideal non-reactive gas are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u} + p\underline{\boldsymbol{\delta}} - \underline{\boldsymbol{\tau}}) = 0, \tag{2}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [E\boldsymbol{u} + (p\underline{\boldsymbol{\delta}} - \underline{\boldsymbol{\tau}}) \cdot \boldsymbol{u} - \kappa \nabla T] = 0, \tag{3}$$

$$\frac{\partial \rho Y_k}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} Y_k) - \nabla \cdot (\rho D_k \nabla Y_k) = 0, \tag{4}$$

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho \boldsymbol{u} \cdot \boldsymbol{u}, \quad p = \rho R T, \tag{5}$$

where ρ is the density, $\boldsymbol{u}$ is the velocity vector, p is the static pressure, E is the total energy, T is the temperature, γ (=1.4:air) is the ratio of specific heats, R is the gas constant, κ is the thermal conductivity, $\underline{\boldsymbol{\delta}}$ is the unit tensor. Eq. (4) is the transport equation for a mass fraction Y_k where D_k is the species diffusion coefficient. The viscous stress tensor $\underline{\boldsymbol{\tau}}$ is

$$\underline{\boldsymbol{\tau}} = \mu(2\underline{\boldsymbol{S}}) + \left(\beta - \frac{2}{3}\mu\right)(\nabla \cdot \boldsymbol{u})\underline{\boldsymbol{\delta}}, \tag{6}$$

where μ is the dynamic (shear) viscosity, β is the bulk viscosity, and $\underline{\boldsymbol{S}}$ is the strain rate tensor, $\underline{\boldsymbol{S}} = \frac{1}{2}(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}})$.

2.2. Localized artificial diffusivity

When a high-order compact scheme is applied to solve flows that involve steep gradients such as those due to shock waves, contact surfaces or material discontinuity, non-physical spurious oscillations that make the simulation unstable