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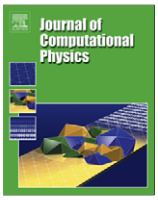

# Front tracking with moving-least-squares surfaces

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A B S T R A C T

The representation of interfaces by means of the algebraic moving-least-squares (AMLS) technique is addressed. This technique, in which the interface is represented by an unconnected set of points, is interesting for evolving fluid interfaces since there is no surface connectivity. The position of the surface points can thus be updated without concerns about the quality of any surface triangulation. We introduce a novel AMLS technique especially designed for evolving-interfaces applications that we denote RAMLS (for Robust AMLS). The main advantages with respect to previous AMLS techniques are: increased robustness, computational efficiency, and being free of user-tuned parameters.

Further, we propose a new front-tracking method based on the Lagrangian advection of the unconnected point set that defines the RAMLS surface. We assume that a background Eulerian grid is defined with some grid spacing $h$. The advection of the point set makes the surface evolve in time. The point cloud can be regenerated at any time (in particular, we regenerate it each time step) by intersecting the gridlines with the evolved surface, which guarantees that the density of points on the surface is always well balanced. The intersection algorithm is essentially a ray-tracing algorithm, well-studied in computer graphics, in which a line (ray) is traced so as to detect all intersections with a surface. Also, the tracing of each gridline is independent and can thus be performed in parallel.

Several tests are reported assessing first the accuracy of the proposed RAMLS technique, and then of the front-tracking method based on it. Comparison with previous Eulerian, Lagrangian and hybrid techniques encourage further development of the proposed method for fluid mechanics applications.

## 1. Introduction

We address in this article the problem of modeling the time evolution of an interface $\mathscr{S}(t)$ which separates two fluids ($A$ and $B$), with the possibility of fluid $B$ being the ambient air (the free-surface case). The problem takes place inside a finite domain $\Omega$, with boundary $\partial\Omega$.

Purely Lagrangian methods for modeling moving interfaces consist of seeding the interface with marker particles and moving the particles as dictated by the velocity field. These methods have proved to be of high accuracy in many published studies [21,45,44,14,35], however with three drawbacks:

- It is difficult to simulate breakup and merging processes of the surface. Topology changes are hard to handle.
- It is difficult to keep the density of particles consistent with the desired level of discretization. They accumulate at some areas while other areas get depleted of particles. Effective particle creation and deletion strategies are needed to handle this issue.

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- Most purely Lagrangian methods maintain the connectivity of the particles [45,44,35], needed to reconstruct the free surface from the scattered particles that move with it. This connectivity defines a mesh of the moving interface. If this mesh gets too distorted the reconstruction becomes unphysical, leading to collapse of the simulation.

The aforementioned drawbacks of Lagrangian methods have made Eulerian methods, such as the volume-of-fluid (VOF) method [20,32] or the level-set (LS) method [30,33,37,27,29,34] (or combinations thereof [36,39]) to be preferred in the modeling of complex interfaces undergoing topological changes such as bubble coalescence, wave breaking, etc. [40,38,47,8,9]. Eulerian methods are based on the advection of a scalar field $\phi$ defined on the whole flow domain. In the VOF method this scalar field represents the partial content of fluid $A$ in each grid cell, whereas in the LS method $\phi$ implicitly defines the interface as its zero-level set. The improved flexibility of Eulerian methods, however, comes at the expense of a loss in accuracy due to interpolation errors, together with numerical errors in the transport of $\phi$.

The previous considerations have motivated the search of numerical methods that combine the accuracy of Lagrangian methods with the flexibility of Eulerian ones. Some developments in this direction are related to the method proposed in this article.

Du et al. [11] combined a purely Lagrangian method with a grid-based reconstruction method which is applied locally in space and time where topological difficulties arise. Since the marker particles have an associated connectivity structure, the reconstruction of tangled regions of the interface is quite sophisticated, especially in three dimensions.

Torres and Brackbill [43] developed a point-set method in which front tracking was performed without a connectivity structure. Their method evolves marker particles according to the velocity field, and then builds a level-set-like function $\phi$ by solving Laplace's equation on $\Omega$. By prescribing $\phi = 1$ at cells containing marker particles and $\phi = 0$ at $\partial\Omega$, the resulting function $\phi$ equals one inside $\mathscr{S}$, identifying the region occupied by fluid $A$. A smoothing procedure based on B-splines, followed by a correction step, are then applied to $\phi$ so that one of its level sets passes through the marker particles. They then regenerate interfacial points as projections of cell centers of an auxiliary finer grid onto the level set of $\phi$.

Enright et al. [12], on the other hand, start from an Eulerian level-set method and improve it by incorporating the information about the location of the interface carried by a set of Lagrangian particles. More specifically, the information is incorporated by merging the interface that results from the Eulerian method with spheres centered at the Lagrangian particles which, at the beginning of the time step, are tangent to the interface. They adopt a surface merging technique that is well-established in computer graphics applications. Their method, known as particle level set (PLS) method, has proved quite successful in many applications [24,29], since topology changes are handled easily by the Eulerian part of the algorithm, which is a level-set method. Further, the PLS method does not require highly accurate Eulerian solvers for the level-set transport equation [13], since the Lagrangian part corrects the inaccuracies. Another method that uses Lagrangian particles to update the interface is the Lagrangian particle level-set method of Hieber and Koumoutsakos [16], in which a method based on smooth particle hydrodynamics (SPH) is implemented.

In this article, we explore the potential of moving-least-squares (MLS) [22] implicit representation of surfaces from point clouds, in the context of modeling interface motion. The proposed method can be viewed as a variant of the front-tracking method of Du et al. [11] without any connectivity of the marker particles. The maker particles follow a Lagrangian motion, but the interface is not assumed to exactly pass through them. Instead, the particles define the interface in an MLS sense. This methodology avoids the cost involved in the linear systems solved by Torres and Brackbill [43] to build $\phi$. Further, from this implicit MLS representation of $\mathscr{S}(t)$ we simultaneously perform the re-generation of points and the construction of a level-set-like function $\phi$ that indicates the region occupied by each fluid. This is done by performing a ray-tracing-like [1,46] detection of $\mathscr{S}(t)$ along the gridlines of a fixed Cartesian mesh $\mathscr{T}_h$. The grid size $h$ of $\mathscr{T}_h$ is the only parameter that governs the approximation of $\mathscr{S}(t)$. The distance between marker particles is automatically kept of order $h$, and features of dimension smaller than $h$ are automatically ignored as in Eulerian methods, but the tracking of the interface remains Lagrangian and thus highly accurate. As compared to the PLS method, on the other hand, the cost of the Eulerian step is avoided and the particles are much less in number and more easily regenerated, since they actually lie on the surface instead of at the centers of tangent spheres.

The plan of this article is as follows: in Section 2 we remind the definitions of implicit algebraic MLS surfaces and perform some convergence tests to evaluate their accuracy. In Section 3 we introduce a parameter-free variant of algebraic MLS surfaces with improved robustness. Section 4 describes the proposed front-tracking algorithm based on MLS surfaces and provides details of its implementation. Extensive numerical testing is reported in Section 5, while Section 6 is devoted to the conclusions.

## 2. Implicit algebraic moving-least-squares surfaces

### 2.1. Basic definitions

Moving-least-squares (MLS) [22] is a method of producing continuous functions from a set of unorganized sampled point values based on the calculation of a weighted-least-squares approximation. In computer graphics, MLS is being used to produce smooth surfaces from point clouds, defining the *MLS surfaces* [4]. Firstly proposed by Alexa et al. [3] as the set of fixed points of the Levin's projection [23], MLS surfaces are becoming a well-established meshless method for modeling and ren-