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Conformal FDTD-methods to avoid time step reduction with and without cell enlargement

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Abstract

During the last decades there have been considerable efforts to develop accurate and yet simple conformal methods for modelling curved boundaries within the finite difference time domain (FDTD) algorithm. In an earlier publication we proposed the uniformly stable conformal (USC) approach as a general three-dimensional extension of FDTD without the need to reduce the maximum stable time step. The main idea of USC is the usage of virtually enlarged cells near to the boundary, leading to an increased implementation effort. In this paper we review the USC method and introduce a new simple and accurate conformal scheme which does not use such enlarged cells. This simplified conformal (SC) scheme has the same number of operations and algorithmic logic as the standard "staircase" method, and thus is easily realizable in existing FDTD codes. Like USC, it leads to accurate results without time step reduction, showing a nearly second order convergence in practice. The method is verified and compared to other approaches by means of several numerical 2D and 3D examples.

Keywords: Maxwell's equations; FDTD; Finite integration; Conformal; Staircase; Wake field

1. Introduction

In the past decades, most of the research on FDTD [1] was focused on overcoming the staircase problem [2,3] of the conventional algorithm. These attempts have resulted in the development of various conformal versions of FDTD [4–16]. However, the most simple and accurate conformal methods (for example, [6,7]) demand to reduce the time step due to the reduction of the effective mesh step sizes near the boundary.

Several years ago, a new stable second order convergent algorithm on Cartesian grids *without time step reducing* was introduced in our paper [16]. The so-called uniform stable conformal (USC) algorithm was described in context of the finite integration technique [17–21] and is based on a conformal scheme introduced

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in [6,7]. The main drawback of the USC algorithm is the usage of extended stencils near to the boundary (or, what is equivalent, exploiting of non-diagonal material matrices).

Motivated by the need for a simplification of USC, both in terms of implementation effort and intuitive understanding, we present in this paper a new simplified conformal (SC) scheme. It does not use extended stencils (or, what is equivalent, exploits only diagonal material matrices), but in the same time it remains accurate and stable without time step reducing. The new scheme is not second order convergent for general geometries. However, it is much more accurate than the "staircase" method. Numerical tests show a second order convergence of the new scheme on moderate meshes. Hence, as shows our experience, in practical examples the scheme has the same level of accuracy as the more complicated USC method. Like the USC scheme the new method is a fully three dimensional technique, with, however, a much simpler realization.

In the first section we shortly describe the finite integration technique (FIT) as the basis for our work, following the contents of the paper [21]. In the second part we review a powerful conformal method, the so-called partially filled cells (PFC) approach, which has been introduced as an extension of FIT in [7]. As an extension of PFC the USC method is reviewed and basic ideas are explained. The new simplified conformal scheme with diagonal material matrices and without time step reducing is introduced in the forth section. Finally, the convergence of the algorithm without the need to reduce the time step is analyzed on several numerical examples in two and three dimensions, and the method is compared with other approaches.

2. The finite integration technique

The conformal methods will be introduced in context of the finite integration technique [17–21].

We consider Maxwell's equations in their integral form on a domain $Q \subset R^3$,

$$\begin{aligned} &\oint_{\partial S} \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \vec{B}\cdot \mathrm{d}\vec{s}, \quad \oint_{\partial S} \vec{H}\cdot \mathrm{d}\vec{l} = \frac{\mathrm{d}}{\mathrm{d}t}\int_S \vec{D}\cdot \mathrm{d}\vec{s} + \int_S \vec{J}\cdot \mathrm{d}\vec{s}, \\ &\oint_{\partial V} \vec{D}\cdot \mathrm{d}\vec{s} = \int_V \rho\, \mathrm{d}v, \quad \oint_{\partial V} \vec{B}\cdot \mathrm{d}\vec{s} = 0, \end{aligned} \tag{1}$$

$(\forall S, V \subset Q)$, with linear, non-dispersive constitutive relations

$$\vec{D} = \varepsilon\vec{E}, \quad \vec{B} = \mu\vec{H}, \quad \vec{J} = \sigma\vec{E}, \quad (\forall x \in Q).$$

Let us start by introducing a grid-based decomposition of the entire computation domain into two dual cell complexes K and $\widetilde{K}$. We concentrate here on a three-dimensional Cartesian mesh, but the complete theory is also applicable to more general mesh types. Unlike in finite difference methods we do not start by allocating field components but rather by allocating the electric voltage along mesh edges and the magnetic flux through mesh cell facets as computational unknowns or state variables respectively:

$$\begin{aligned} &\widehat{e}_\vartheta = \int_{L_\vartheta} \vec{E}\cdot \mathrm{d}\vec{l}, \quad \widehat{h}_\vartheta = \int_{\widetilde{L}_\vartheta} \vec{H}\cdot \mathrm{d}\vec{l}, \\ &\widehat{\widehat{d}}_\vartheta = \int_{\widetilde{S}_\vartheta} \vec{D}\cdot \mathrm{d}\vec{s}, \quad \widehat{\widehat{b}}_\vartheta = \int_{S_\vartheta} \vec{B}\cdot \mathrm{d}\vec{s}, \quad \widehat{\widehat{j}}_\vartheta = \int_{\widetilde{S}_\vartheta} \vec{J}\cdot \mathrm{d}\vec{s}, \end{aligned}$$

where ϑ is a mesh multi-index, and $L_\vartheta, S_\vartheta \in K$, $\widetilde{L}_\vartheta, \widetilde{S}_\vartheta \in \widetilde{K}$ are the edges and facets of the primary and dual mesh, respectively. Solving the first Maxwell equation in integral form for the surface shown in Fig. 1(a) yields:

$$\widehat{e}_{xijk} + \widehat{e}_{yi+1jk} - \widehat{e}_{xij+1k} - \widehat{e}_{yijk} = -\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\widehat{b}}_{zijk}.$$

with $i = 1..N_x, \quad j = 1..N_y, \quad k = 1..N_z$. Note, that this representation is still exact, as the $\widehat{e}_\vartheta$ are (by definition) the exact electric voltages along the edges of the surface, and similarly the $\widehat{\widehat{b}}_\vartheta$ represent the exact value of the magnetic flux density integral over the cell surface. (Note also the orientations in coordinate directions.)

If we compose column vectors $\widehat{\mathbf{e}}$ and $\widehat{\widehat{\mathbf{b}}}$ out of all voltage- and flux-components, we can write the combination of all equations over all surfaces in an elegant matrix form as