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Outflow boundary conditions for the Fourier transformed three-dimensional Vlasov–Maxwell system

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Abstract

A problem with the solution of the Vlasov equation is its tendency to become filamented/oscillatory in velocity space, which in numerical simulations can give rise to unphysical oscillations and recurrence effects. We present a three-dimensional Vlasov–Maxwell solver (three spatial and velocity dimensions, plus time), in which the Vlasov equation is Fourier transformed in velocity space and the resulting equations solved numerically. By designing absorbing outflow boundary conditions in the Fourier transformed velocity space, the highest Fourier modes in velocity space are removed from the numerical solution. This introduces a dissipative effect in velocity space and the numerical recurrence effect is strongly reduced. The well-posedness of the boundary conditions is proved analytically, while the stability of the numerical implementation is assessed by long-time numerical simulations. Well-known wave-modes in magnetized plasmas are shown to be reproduced by the numerical scheme.

Keywords: Vlasov–Maxwell system; Fourier method; Outflow boundary

1. Introduction

The most common method to solve the Vlasov equation is with the particle-in-cell (PIC) method [1,2]. In this method, the Vlasov equation is solved by following the trajectories of a set of statistically distributed super-particles, which resolves the particle distribution functions in phase space. Each super-particle represents a large number of real particles. PIC simulations have proven to be extremely successful due to their relative simplicity and adaptivity, especially in problems involving large-amplitude waves and beams. However, the statistical noise of PIC simulations sometimes overshadows the physical results, and for some problems, the low-density velocity tail of the particle distribution cannot be resolved with high enough accuracy by the super-particles.

As a contrast, grid-based Vlasov solvers discretizes the particle distribution function directly in the form of a phase fluid which is represented on a grid in both space and velocity (or momentum) space. The advantage

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with grid-based Vlasov solvers is that there is no statistical noise in the simulations, and that the dynamical range is determined by the number system of the computer rather than super particles. Hence the low-density velocity tail of the particle distribution can be resolved much more accurately by grid-based Vlasov solvers, which makes them suitable for certain types of problems. A disadvantage with grid-based Vlasov solvers in higher dimensions is that the full phase-space has to be discretized on a grid, which makes both the storage of the data in the computer's memory, and the numerical calculations, extremely demanding. These problems are today treated most efficiently by well known PIC codes by enhancing the number of macro-particles. Another problem is the tendency of the distribution function to become oscillatory in velocity space, leading to unphysical noise and recurrence effects in the numerical solution. This was recognized in early Vlasov simulations [3] and special methods were devised to resolve this problem, for example in the time-spitting method [4,5] where a smoothing operator was applied to remove the highest oscillations in velocity space. The time-splitting method has been extended to multiple dimensional Vlasov–Maxwell system where a Van Leer type of dissipative scheme was used [6]. A scheme based on finite volume conservative discretization has also been developed [7], in which an upwind scheme with second-order flux limiter was used to reduce unphysical oscillations in velocity space. A back-substitution method, which avoids artificial heating of the plasmas, has been designed for the integration of the Vlasov equation in a magnetic field [8]. Several Eulerian grid-based solvers are reviewed and compared in Refs. [9,10]. For the Fourier-Fourier method, in which the Vlasov equation is Fourier transformed in both configuration space and velocity space, a filtered method was developed based on a convolution by a Gaussian function in velocity space [11,12], leading to a smoother solution. The Fourier transformed Vlasov equation was also studied both theoretically and numerically to give a new interpretation of Landau damping, the time echo phenomenon etc., in terms of imperfectly trapped (leaking) waves in the Fourier transformed velocity space [13,14]. For methods using Hermite polynomials to resolve the velocity space, methods have been developed in which the highest-order Hermite polynomials absorb the oscillations in velocity space [15,16], thereby reducing numerical recurrence effects strongly. For the Fourier method in one and two dimensions [17–19], absorbing boundary conditions were designed at the largest Fourier mode in velocity space so that the highest oscillations in velocity space were removed from the solution. In this manner, the recurrence effect could be strongly reduced, compared to the naive method of setting the highest Fourier component to zero.

In this article, we extend the algorithms for solving the Fourier transformed one- and two-dimensional Vlasov–Maxwell system [17–19] to three velocity and spatial dimensions, plus time. The absorbing boundary conditions for the Vlasov equation in the Fourier transformed velocity space is proved to be well-posed, and we demonstrate by long simulations with random numbers as initial conditions that there are no numerical instabilities in the system. Several simulations are performed to assess that the numerical scheme reproduces known wave modes in magnetized plasmas.

The article is organized in the following fashion. In Section 2 the three-dimensional Vlasov–Maxwell system is discussed, together with the Fourier transform technique in velocity space. Well-posed boundary conditions are derived in preparation for the numerical simulation of the Fourier-transformed system. The numerical schemes used to approximate the time-dependent solution of the Vlasov equation system are described in Section 3, and the numerical experiments and results are presented in Section 4, where the results are compared with known theory. In Section 5 some conclusions are drawn and future perspectives discussed.

2. The Vlasov–Maxwell system

The Vlasov equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}} f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}} f_\alpha = 0, \tag{1}$$

describes the evolution of the distribution function $f_\alpha(\mathbf{x}, \mathbf{v}, t)$ of electrically charged particles of type α (here α equals i for ions and e for electrons) in the presence of the electric and magnetic field $\mathbf{E}$ and $\mathbf{B}$, respectively, each particle having the electric charge q_α and mass m_α. One Vlasov equation is needed for each species of particles. Below, we will assume electrons and one species of singly charged ions so that $q_e = -e$ and $q_i = -e$ where e is the magnitude of the electron charge.