Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Physics 221 (2007) 626–648
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

Numerical simulation of the fluid dynamics of 2D rigid body motion with the vortex particle method

Jeff D. Eldredge *

Mechanical and Aerospace Engineering, University of California, Los Angeles, 420 Westwood Plaza, Box 951597, Los Angeles, CA 90095-1597, United States

Received 22 August 2005; received in revised form 23 June 2006; accepted 23 June 2006
Available online 14 August 2006

Abstract

A viscous vortex particle method is presented for computing the fluid dynamics of two-dimensional rigid bodies in motion. The Navier–Stokes equations are solved using a fractional step procedure. Smooth particles carry vorticity and exchange strength to account for convection and viscous diffusion. The spurious slip resulting from this half-step is identified with a surface vortex sheet, and the slip is eliminated by diffusing the sheet to adjacent particles. Particles are remeshed every few time steps to a Cartesian grid with a 'body-ignorant' interpolation using simple symmetric stencils. Kelvin's circulation theorem remains enforced by accounting for the circulation leaked into the body during this procedure, and redistributing it to the particles in the subsequent sheet diffusion. The stability and convergence with respect to numerical parameters are explored in detail, with particular focus on the residual slip velocity. The method is applied to two problems that demonstrate its utility for investigating biological locomotion: a flapping elliptical wing with hovering insect kinematics, with good agreement of forces with previous simulations and experiments; and a three-linkage 'fish' undergoing undulatory mechanics.

MSC: 65M99; 76M23; 76D25; 76Z10

Keywords: Numerical methods; Vortex methods; Viscous incompressible flow; Flow–structure interaction; Biological locomotion; Immersed boundary methods

1. Introduction

The interaction of a fluid with a complex moving boundary arises frequently in mechanics, particularly in natural and biological systems: insect flight, aquatic locomotion, cardiopulmonary flows, flows through the human vocal tract, and countless others. Investigations are enhanced greatly by a complementary computational effort, particularly because of the difficulty of obtaining in vivo flow measurements in these systems.

* Tel.: +1 310 206 5094; fax: +1 310 206 4830.
E-mail address: eldredge@seas.ucla.edu.

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2006.06.038

However, conventional computational approaches that rely on body-conforming grids are challenged by such evolving geometries. It is useful, then, to seek an alternative numerical methodology that is more naturally suited to such problems.

In this work, we are primarily motivated by investigations of the biological mechanics of mobility, such as used by insects, aquatic animals and bioinspired technological systems, in a moderate Reynolds number flow regime. Viscous and inertial processes are both important in these forms of locomotion, and vortex shedding plays a fundamental role. A complete solution of the fluid dynamics of such problems requires a three-dimensional unsteady simulation, fully coupled with a solution of the elastically deforming body. Therefore, the numerical approach taken must be capable of such challenging requirements. This paper, which demonstrates the use of the viscous vortex particle method, represents a first step towards this goal: the analysis of two-dimensional rigid bodies in motion. By focusing on the creation and subsequent transport and diffusion of vorticity, this method provides a unique perspective and a useful alternative to other velocity-based numerical methodologies. The velocity field is recovered from vorticity-bearing particles by the Biot–Savart integral, so boundary conditions at infinity are automatically enforced. The method possesses an inherent economy in the number of computational elements since vorticity is generally confined to regions near the bodies and in their wake. Though the focus of this present work is on two-dimensional rigid bodies, the method will be extended to deformable and three-dimensional problems in later work.

Compared to the rich history of experimental investigation of insect flight and aquatic locomotion, computational studies have only recently reached maturity. As is common with complex physical systems, research in this field has focused on progressively more difficult 'building-block' problems to advance the state of knowledge. For example, much has been learned from the basic flow produced by the pitching and plunging of a two-dimensional rigid foil, which may be representative of a section of a high aspect ratio wing. In this system, the time-varying translational velocity and angle of attack can be imposed via the free-stream velocity, allowing a stationary, body-fitted computational grid. This approach was taken by Gustafson and Leben [1], and more recently by Wang [2].

Several computational studies of greater complexity have also relied on a traditional body-fitted grid methodology. Sun and Tang [3] used a time-dependent, body-conforming grid to obtain a three-dimensional solution for the flow around a fruitfly wing, to compare with the mechanical wing measurements of Dickinson, Lehmann and Sane [4]. The Navier–Stokes equations, expressed in an inertial Cartesian coordinate system, were transformed to a body-fitted curvilinear system using a mapping that varied with time as the wing configuration changed. A similar approach was used by Liu and Kawachi [5] to investigate the aerodynamics around the wing of a hawkmoth, and compare with delayed stall experimental results [6]. A hawkmoth wing consists of fore and hind portions attached along a common axis, and an allowance for changes in the relative angle-of-attack between these portions presented an additional challenge for the grid algorithm. Numerical studies of the fruitfly wing were also conducted by Ramamurti and Sandberg [7], using a finite element solution of the governing equations in arbitrary Lagrangian–Eulerian (ALE) formulation. The ALE approach allows the near-wing grid to move as the wing does, but remeshing is still required to eliminate badly distorted elements.

Though such body-fitted approaches allow a direct enforcement of the boundary conditions, the grid generation is challenging, particularly for the large motions and deformations characteristic of insect flight. Thus, methods in which the boundary is immersed in a regular Cartesian grid have been a particularly active area of research. In these methods, the effect of the boundary is either imposed directly—via interpolation of the surface velocities to nearby grid points—or indirectly, through some sort of singularity distribution (such as forcing terms or a vortex sheet). In the context of insect flight, both artifices have been used. The finite volume approach developed by Udaykumar and Mittal and coworkers [8,9] is an example of a direct Cartesian grid method, in which cells near the interface are cut to enforce a sharp boundary. The method has been used to solve for the fluid motion produced by a pair of two-dimensional flapping wings [10]. A notable example of an indirect approach is the immersed boundary method of Peskin and coworker [11,12], wherein an elastic boundary is represented by the singular distribution of forces it applies to the fluid. The singularity is smoothed in order to transfer its effect from the boundary to the grid. Beyond its extensive use in heart mechanics, the method has been applied to swimming organisms [13], and recently to insect hovering [14]. Hybrid methods that exploit the advantages of a sharp interface and the convenience of a fixed grid have been