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Face offsetting: A unified approach for explicit moving interfaces

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Abstract

Dynamic moving interfaces are central to many scientific, engineering, and graphics applications. In this paper, we introduce a novel method for moving surface meshes, called the *face offsetting method*, based on a generalized Huygens' principle. Our method operates directly on a Lagrangian surface mesh, without requiring an Eulerian volume mesh. Unlike traditional Lagrangian methods, which move each vertex directly along an approximate normal or user-specified direction, our method propagates faces and then reconstructs vertices through an eigenvalue analysis locally at each vertex to resolve normal and tangential motion of the interface simultaneously. The method also includes techniques for ensuring the integrity of the surface as it evolves. Face offsetting provides a unified framework for various dynamic interface problems and delivers accurate physical solutions even in the presence of singularities and large curvatures. We present the theoretical foundation of our method, and also demonstrate its accuracy, efficiency, and flexibility for a number of benchmark problems and a real-world application.

Keywords: Interface propagation; Moving meshes; Entropy condition; Huygens' principle; Face offsetting

1. Introduction

Dynamic moving surfaces arise in many scientific and engineering applications, such as crystal growth, dendritic solidification, microchip fabrication, multiphase flows, combustion, and biomedical applications. Computer simulations of these applications require numerical techniques to determine the position of a surface (or interface) as it evolves under a speed or velocity field governed by some physical laws. This problem is commonly referred to as *interface propagation*. Because singularities and topological changes may develop during the evolution of the surface, this problem poses daunting challenges in developing accurate, efficient, and robust techniques, and has attracted significant interests in the past two decades [1–11].

In general, there are two broad categories of techniques for moving interfaces: *Lagrangian methods*, which *track* the interface using an *explicit* surface representation, and *Eulerian methods*, which *capture* the interface using an *implicit* representation. In recent years, Eulerian methods (such as level set methods [1–3], volume-

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of-fluid methods [12–15], and phase-field methods [16]) have made significant advancements and become the dominant methods for moving interfaces for their simplicity and robustness, although their accuracy may be low in the presence of singularities and topological changes. Lagrangian methods, such as marker particle [17] and front tracking methods [4–6], can potentially provide higher accuracy at lower cost and frequently suit better than implicit representations for applications such as multi-component simulations. Unfortunately, existing Lagrangian methods often suffer from instabilities in the form of growing oscillations or self-intersections [3] and sometimes must resort to *ad hoc* techniques to trim off self-intersections [18,19]. Due to this mixed success of pure Lagrangian or Eulerian methods, in an attempt to seek a better compromise between the accuracy of Lagrangian methods and robustness of Eulerian methods, a number of *hybrid methods* have appeared in recent years [7–10]. These methods unfortunately are more complex to implement than traditional methods and still suffer from smearing of singularities (although milder than pure Eulerian methods).

In this paper, we introduce a novel method for moving interfaces, called the *face-offsetting method* (FOM). As a Lagrangian method, FOM propagates an explicit surface mesh without requiring a volume mesh. Unlike traditional Lagrangian methods, which moves each vertex passively using some standard numerical integration techniques, FOM propagates the faces of the mesh and then reconstructs the vertices from the propagated faces by performing an eigenvalue analysis locally at each vertex to resolve the normal motion (for obtaining surface geometry) and tangential motion (for maintaining mesh quality) simultaneously. This new approach is motivated by a new formulation of interface propagation, referred to as the *generalized Huygens' principle*, which extends the well-known *entropy-satisfying Huygens' principle* behind the level set methods [3]. This new principle explores some underlying connections between traditional Lagrangian and level set formulations of moving interfaces, unifies the view for *advective* motion (such as rigid-body motion and transport of fluids) and *wavefrontal* motion (such as burning, erosion, and deposition), and in turn provides us a new foundation for Lagrangian methods to obtain accurate physical solutions for both types of motions even at singularities, and to ensure the integrity of the surface as it evolves.

The remainder of this paper is organized as follows. Section 2 revisits some fundamental concepts behind interface propagation and introduces the generalized Huygens' principle. Section 3 introduces the face offsetting method for advective motion, and Section 4 generalizes it to wavefrontal motion. Section 5 reports some experimental results using benchmark problems and a real-world application.

2. Unified view of moving interfaces

2.1. Moving interfaces

Dynamic surfaces appear in different forms in a wide range of applications. In this paper, we will focus on *moving interfaces*, where an interface refers to an *orientable* surface Γ^0 (or Γ), separating its *inside* Γ^+ from its *outside* Γ^-. The problem that we address in this paper, referred to as *interface propagation*, is to determining the position of a moving interface at a specific time, given its position and either a *velocity field* $\dot{\boldsymbol{u}}(\boldsymbol{x},t) : \Gamma \times \mathbb{R} \to \mathbb{R}^3$ or a *normal speed* $f(\boldsymbol{x},t) : \Gamma \times \mathbb{R} \to \mathbb{R}$. In the traditional Lagrangian formulation, the differential equation for propagating an interface under a given velocity is

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \dot{\boldsymbol{u}}(\boldsymbol{x},t), \tag{1}$$

and under a given normal speed is

$$\frac{\partial \boldsymbol{x}}{\partial t} = f\boldsymbol{n}(\boldsymbol{x},t), \tag{2}$$

where $\boldsymbol{n}$ is the unit surface normal $\partial\boldsymbol{x}/\partial u \times \partial\boldsymbol{x}/\partial v / \|\partial\boldsymbol{x}/\partial u \times \partial\boldsymbol{x}/\partial v\|$ given a 2-D unit-length orthogonal parameterization $\boldsymbol{x}(u,v) : \mathbb{R}^2 \to \mathbb{R}^3$. In principle, a given velocity field can be converted into a normal speed and vice versa, as a tangential motion does not change the shape of the interface. However, propagation under a normal speed is in general more difficult than under a velocity field because the normal direction must be inferred from the geometry of the moving interface.