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Journal of Computational Physics 217 (2006) 217–247

JOURNAL OF COMPUTATIONAL PHYSICS

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Uncertainty quantification of limit-cycle oscillations ☆

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Received 9 September 2005; received in revised form 29 March 2006; accepted 30 March 2006
Available online 30 June 2006

Abstract

Different computational methodologies have been developed to quantify the uncertain response of a relatively simple aeroelastic system in limit-cycle oscillation, subject to parametric variability. The aeroelastic system is that of a rigid airfoil, supported by pitch and plunge structural coupling, with nonlinearities in the component in pitch. The nonlinearities are adjusted to permit the formation of a either a subcritical or supercritical branch of limit-cycle oscillations. Uncertainties are specified in the cubic coefficient of the torsional spring and in the initial pitch angle of the airfoil. Stochastic projections of the time-domain and cyclic equations governing system response are carried out, leading to both intrusive and non-intrusive computational formulations. Non-intrusive formulations are examined using stochastic projections derived from Wiener expansions involving Haar wavelet and B-spline bases, while Wiener–Hermite expansions of the cyclic equations are employed intrusively and non-intrusively. Application of the B-spline stochastic projection is extended to the treatment of aerodynamic nonlinearities, as modeled through the discrete Euler equations. The methodologies are compared in terms of computational cost, convergence properties, ease of implementation, and potential for application to complex aeroelastic systems.
Published by Elsevier Inc.

Keywords: Uncertainty quantification; Stochastic expansion; Polynomial chaos expansion; Harmonic balance; Limit cycle oscillation; Aeroelastic

1. Introduction

The desire to simulate limit-cycle oscillation (LCO) in aeroelastic systems has become increasingly practical over the last decade. While many significant challenges yet remain in modeling the phenomenon in a reliable and verifiable manner, there is little doubt that some of the key computational elements have taken shape. Recent work by Thomas et al. [36] and Farhat et al. [11] demonstrate that complex aeroelastic responses

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0021-9991/$ - see front matter. Published by Elsevier Inc.
doi:10.1016/j.jcp.2006.03.038

can be captured with Euler and Navier–Stokes analysis for geometry of practical significance. Additional work by Beran et al. [4] and Denegri and Dubben [8] exemplify the degree to which simulation with transonic small-disturbance theory can yield additional insights into the phenomenology of LCO.

Thomas et al. have commented on the sensitivity of computed LCOs to the modeled values of vehicle properties in their models [37]. They observed that small changes (<5%) in the natural frequencies of structural modes participating in the LCO of an F-16 caused large changes in LCO amplitude (>30%) and decrease of the speed of LCO onset (≈5%). This latter change, while seemingly small, represents a substantial and surprising reduction in the modeled operational capability of the vehicle. Their findings may indeed be a reflection of sensitivity of the actual phenomenon to real-world variations in the aeroelastic system. Currently, there exists a substantial flight-test program for the F-16 that helps to certify the vehicle for flight safety with any possible store configuration (i.e., externally mounted tank or munition) [5]. In many ways, the existence of this test program is a testimony to the sensitive degree to which the F-16's aeroelastic behavior depends on store properties such as weight, location, geometric shape (with or without fins), and airframe linkage. Within the test and evaluation community, there is much anecdotal knowledge concerning the variability in LCO response characteristics observed for fighter aircraft. An experimental study reported by Cunningham [6] examined how nominally identical aircraft could experience different aeroelastic responses based on variations in horizontal-tail structure within manufacturing tolerances.

Owing to the practical importance of avoiding or limiting LCO in operational vehicles, it is sensible to study the generic problem of computing the dependence of nonlinear oscillations on variations in system parameters. This effort serves to highlight some of the key computational issues through examination of idealized problems exhibiting LCO, and provides a roadmap for developing a more advanced capability suitable for real-world configurations.

While the literature is relatively rich in the stochastic analysis of problems that are either static, linear, or both, there is little work directed towards describing nonlinear processes that are dynamic with compact stochastic representations. Certainly, LCOs represent only one sub-class of dynamic processes; but they represent an important category of autonomous solutions that bifurcate from systems otherwise in equilibrium, and are the subject of study in many fields outside of the aerospace sciences. Furthermore, LCOs are challenging to simulate, in that the physical times needed to realize fully developed responses can be quite large. As it will be seen, this challenge is magnified when systems are analyzed stochastically.

Restricting the present review of stochastic analysis to those related to LCOs, comments on noteworthy features of several articles can be made. In a foundational effort, Xiu et al. [40] analyzed the stochastic response of a structurally supported cylinder in crossflow, subject to variability in structural stiffness. Assuming a Gaussian probability density function (PDF) for the random parameter, they computed PDFs of cylinder position at different time levels using Wiener expansions of the dependent variables in the specified random variable. These expansions will be described later, but yield a spectral (i.e., efficient) means for associating system response with values of input parameters, e.g., those selected in a sampling process. Stochastic solutions were tracked into the development of LCO, an aeroelastic phenomenon sustained in this problem by vortex shedding. Millman et al. [25] studied the LCO of a structurally supported airfoil in the time domain using modeled aerodynamics and a new Wiener expansion of the stochastic response with improved convergence properties. Focusing on the bifurcation characteristics of the system, nonlinearity of the torsional support was adjusted to yield a subcritical Hopf bifurcation, which enabled bi-modal responses to be analyzed.

In a set of papers in 2004, the authors and their colleagues studied the large-time failure of Wiener expansions in the time domain, and examined uniformly convergent means for characterizing uncertainty in LCO responses. First, Pettit and Beran [30] critically examined why stochastic analysis in the time-domain fails, using the previous airfoil problem. They found that the nonlinearity of the stochastic projection increases in time, such that any fixed projection becomes unsatisfactory in capturing the nonlinearity at a sufficiently large time. Beran and Pettit [2] proposed a non-time-domain approach to capturing LCO that is rapidly convergent, describing the stochastic behavior of the airfoil response in a very small number of orbital modes. In their work, one structural parameter, the cubic stiffness coefficient, was considered random. Millman et al. [26] alternatively proposed a stochastic projection method derived from B-splines and apply the technique to representing the probabilistic response of the airfoil in terms of a single output variable, the peak pitch angle. Two input variables were considered random, the cubic stiffness coefficient and the initial angle of attack. They