Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Computational Physics 211 (2006) 385–404
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

The repair paradigm: New algorithms and applications to compressible flow

Raphaël Loubère *, Martin Staley, Burton Wendroff

Los Alamos National Laboratory, Theoretical Division T-7, MS B284, Los Alamos, NM 87545, USA

Received 14 October 2004; received in revised form 5 April 2005; accepted 20 May 2005

Abstract

The repair paradigm leads to several algorithms for redistributing mass, momentum and energy, while adhering to local maximum principles, as an adjunct to the remapping step in certain compressible flow codes that use remapping, such as Arbitrary–Lagrangian–Eulerian codes, or for just redistributing mass in advection codes. In the case of advection of a concentration, repair keeps the newly computed concentration in a cell between the maximum and minimum concentrations in neighboring old cells, thus guaranteeing at least that the new concentration is between zero and one. For compressible flow, density, velocity and internal energy are similarly constrained while maintaining conservation of mass, momentum and total energy. In this way, positive density and internal energy are achieved as a side effect. We propose a new algorithm, combining both local and global repair, that maintains causality and is efficient in a parallel computational setting. The local/global algorithm is independent of the order in which the distribution is performed, and it maintains 1D symmetry. This is applied to advection in two dimensions, and to, among others, the LeBlanc problem, the Sedov problem, and an interacting 2D blast wave problem. The latter is done with a Lagrangian code for which rezoning, remapping and repair are essential.

Keywords: Repair; Mass redistribution; Conservative reconstruction; Remapping; Advection; Hydrodynamics

1. Introduction

A critical part of Lagrangian-based methods for computational fluid dynamics (CFD) is the ability to remap or interpolate data from one computational mesh to another. This is the case for the popular

* Corresponding author. Tel.: +1 505 667 1407.
E-mail addresses: loubere@lanl.gov (R. Loubère), mstaley@lanl.gov (M. Staley), bbw@lanl.gov (B. Wendroff).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2005.05.010

ALE schemes that perform Lagrangian steps followed by remaps to fixed grids. Remapping is also essential for pure Lagrangian methods, because they can lead to tangled grids that must then be untangled with a concomitant remap step. Even if the basic scheme produces only physically meaningful quantities, a remapping method can create out-of-bounds quantities such as negative densities or pressures. In some CFD codes, the offending values are simply set to a small positive number when this occurs, at which point mass or total energy is no longer conserved. In most instances the error thereby created is negligible, but we shall show that in at least one example the error is significant.

It is possible, by taking great care with the remapping in the CFD context, to maintain positive mass density. This is done by first extending the given mean densities in each original cell to the whole domain so that the new distribution is everywhere positive, and then computing new mean values by exact integration over the cells of the new grid. Total energy can be remapped in this way, but then there is no guarantee that internal energy will be positive. Furthermore, in more than one dimension, exact integration is computationally intensive.

Another context in which non-physical data can occur is in divergence-free advection of a concentration that must retain values between zero and one. High quality advection schemes, some of which are based on remapping ideas [1,2], unavoidably have this fault [3].

The goal in this paper is to improve upon and apply the repair idea introduced in [4,5]. A repair method can be viewed as a way to correct values on a discrete mesh by redistributing the conserved quantity so that conservation and a maximum principle are preserved. The maximum principle is that new values should obey certain upper and lower bounds obtained from old values. In this way, not only are non-physical quantities eliminated, but oscillations are reduced (albeit not necessarily eliminated). We therefore seek repair algorithms that can be applied to CFD problems, advection problems, or other situations where values of a discrete variable must be placed in bounds without violating a conservation law and without introducing significant errors in the dynamics.

As stated in [4] (Section 8, p275), repair is a mass redistribution nonlinear filter. Other methods for the correction of nonphysical data, such as flux corrected transport, are discussed in [3].

The rest of this paper is arranged as follows. We first present notation, goals and expected properties of repair methods. We then review a local repair method [5] which repairs out-of-bounds values and distributes the resulting mass discrepancies locally. This method can produce different results depending on the order in which cells are visited, and it is therefore called order-dependent. Next we review a simple global repair process [4] which repairs out-of-bounds values and distributes the resulting mass discrepancy across the entire grid. The next two sections introduce order-independent local methods, and we conclude with a discussion of repair methods in advection and hydrodynamics contexts, where numerical tests are performed to show the effects of such methods.

2. Notation, goals and properties

Repair methods can be used for many kinds of variables, including density, velocity, energy, pressure, and concentration, but we will henceforth call our variable to be repaired a density ρ, or equivalently, a mass m. If we denote old cells by c and new cells by $\tilde{c}$, then the quantity to be conserved is the total mass $m = \sum_c m(c) = \sum_c \rho(c)V(c)$, where $m(c)$, $\rho(c)$, and $V(c)$ denote the mass, density, and volume, respectively, of cell c.

Consider an old mesh $\mathscr{M}$ with cell-averaged densities (called old densities), and a new mesh $\widetilde{\mathscr{M}}$ with remapped cell-averaged densities (called new densities). We assume for simplicity that the connectivity is the same for the old and new grids.

In the case of advection the meshes would coincide, but typically the new mesh is a small perturbation of the old one. Define the *bound* neighborhood $N(c)$ of a cell c as a patch of surrounding cells,