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AEGIS: An adaptive ideal-magnetohydrodynamics shooting code for axisymmetric plasma stability

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Abstract

A new linear ideal-magnetohydrodynamics stability code for axisymmetric plasmas, AEGIS, is described. The AEGIS code employs adaptive shooting in the radial direction and Fourier decomposition in the poloidal direction. The general solution is a linear combination of the independent solutions of the Euler–Lagrange equations solved by the adaptive shooting. A multiple-region matching technique is used to overcome the numerical difficulty associated with the stiff nature of the independent solutions. Benchmarks with other MHD codes show good agreement. Because it is adaptive, the AEGIS code has very good resolution near the singular surfaces of MHD modes. AEGIS has the additional advantage of allowing the investigation of modes with not only low mode numbers, but also intermediate to high mode numbers.

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1. Introduction

Successful magnetic confinement of a toroidal fusion plasma relies on the magnetohydrodynamic (MHD) stability of the plasma. Due to the complexity of toroidal geometry, numerical computation of the MHD instability modes in toroidal plasmas is indispensable for interpreting experimental observations and designing new devices. During the last 30 years, several MHD stability codes have been developed, such as PEST, ERATO, GATO, DCON, NOVA, KINX, MISHKA, etc. [1–11].

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The MHD eigenvalue-type codes – for example, PEST [1,2,6] and GATO [5] – usually employ the finite element method with a fixed grid for solving the radial eigenvalue problem. As is well known, the ideal MHD eigenvalue problem can be singular at mode rational surfaces in the case of marginal stability. To resolve this singular feature, an adaptive grid mesh in the radial dimension is desirable. The DCON code [7] employs an adaptive method; however, it uses the generalized Newcomb theorem [12] to determine stability of the internal modes and computes only the marginal stability of both internal and external MHD modes.

The current paper describes a newly developed adaptive eigenvalue shooting code for ideal MHD equations that can solve the finite frequency (or growth rate) eigenvalue problem. The finite frequency problem needs to be addressed in incorporating rotational and kinetic effects and in studying the Alfvén modes. Therefore, the numerical scheme presented in this article has the potential for being directly extended beyond low-frequency ideal MHD computations. The new code is named AEGIS (Adaptive EiGenfunction Independent Solution). The AEGIS code uses Fourier decomposition in the poloidal direction. To solve the radial eigenvalue problem, it constructs the general solution through a linear combination of the independent solutions of the Euler–Lagrange equations. The eigenvalue problem is then formulated by this general solution being fitted to the boundary conditions at the magnetic axis and the plasma-vacuum interface. This formulation is similar to that employed in the DCON code for marginal stability and in the ELITE code for high-mode-number MHD modes [13–15]. The difference from DCON is that AEGIS addresses modes with finite frequency or growth rate. The difference from ELITE is that AEGIS addresses both low- and intermediate-to-high-mode-number modes. To compute the eigenvalue problem with finite frequency (or growth rate), the modes at the singular layers must to be resolved. This poses a difficulty in applying the method based on decomposition in terms of the independent solutions. Although the MHD eigenmodes are well-behaved, the independent solutions are generally very stiff because near marginal stability the MHD modes are singular at the mode resonance surfaces, as well as at the magnetic axis. Hence, continuously shooting from the magnetic axis to the plasma edge usually fails to obtain accurately the independent solutions that extend across the whole plasma. The small solution of the Euler–Lagrange equation is often submerged in the numerical noise of the large solution. To resolve this difficulty, the AEGIS code employs a multiple-region matching technique. In this procedure, shooting in the radial dimension is performed in multiple regions, and then the independent solutions obtained in the individual regions are matched to each other. Note that the DCON code uses the Gaussian elimination technique to remove the large solution in order to resolve the numerical difficulty due to singularity at resonance surfaces. We find that our numerical scheme in AEGIS can effectively handle the ideal-MHD eigenvalue problem. It therefore opens the door for the extension to kinetic MHD computations, which in general need to deal with the MHD large solution.

The paper is arranged as follows: In Section 2, the equilibrium calculation is described. In Section 3, the numerical scheme for determining the independent solutions is detailed. The eigenvalue problem for determining the stability is then formulated with the use of these independent solutions. In Section 4, the numerical results and the benchmarks with the GATO code are described. In the last section, the results are discussed and conclusions are presented.

2. Equilibrium

In a toroidally symmetric configuration, the magnetic field $\vec{B}$ can be expressed as

$$\vec{B} = \chi' \nabla\phi \times \nabla\psi + g(\chi)\nabla\phi, \tag{1}$$

where ϕ is the axisymmetric toroidal angle, ψ labels the magnetic surface, $\chi(\psi)$ denotes the poloidal magnetic flux, $g(\chi)$ is the poloidal current flux and a prime denotes a derivative with respect to ψ. An overhead arrow is used to indicate vectors in configuration space. The poloidal flux χ is governed by the Grad-Shafranov equation, with the pressure profile $P(\chi)$ and poloidal current flux $g(\chi)$ to be specified. The