Journal of Informetrics 7 (2013) 737–745

Contents lists available at ScienceDirect

Journal of Informetrics

journal homepage: www.elsevier.com/locate/joi

Mathematical properties of Q-measures

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ARTICLE INFO

Article history:
Received 16 May 2013
Received in revised form 13 June 2013
Accepted 17 June 2013
Available online 11 July 2013

Keywords:
Network theory
Q-measures
Betweenness centrality

ABSTRACT

Q-measures are network indicators that gauge a node's brokerage role between different groups in the network. Previous studies have focused on their definition for different network types and their practical application. Little attention has, however, been paid to their theoretical and mathematical characterization. In this article we contribute to a better understanding of Q-measures by studying some of their mathematical properties in the context of unweighted, undirected networks. An external Q-measure complementing the previously defined local and global Q-measure is introduced. We prove a number of relations between the values of the global, the local and the external Q-measure and betweenness centrality, and show how the global Q-measure can be rewritten as a convex decomposition of the local and external Q-measures. Furthermore, we formally characterize when Q-measures obtain their maximal value. It turns out that this is only possible in a limited number of very specific circumstances.

1. Introduction

Many research topics are nowadays studied from a network perspective. This is not only the case in the life sciences and the physical sciences, but also in the social sciences and the humanities (Goh et al., 2007; Newman, 2003; Risse, 2000; Wasserman & Faust, 1994). Large-scale analyses of so-called complex networks reveal that the same structural features – such as skewed degree distributions and local clustering – can emerge in different fields. This underlines the importance of network studies.

The field of informetrics is no exception to this trend. Indeed, topics such as collaboration, diffusion and citation have been studied from the perspective of social network analysis (see e.g., Franceschet, 2012; Leydesdorff, 2007; Liu, Rafols, & Rousseau, 2012; Liu, Rousseau, & Guns, 2013; Otte & Rousseau, 2002; Rousseau, Liu, & Ye, 2012; Yan & Ding, 2009). In these studies special attention is often paid to ranking entities (authors, journals, papers, etc.) according to one or more network indicators. The simplest example is ranking by (in-)degree; in the case of citation networks this is equivalent to ranking by number of citations. Moreover, the field of social network analysis (SNA) has introduced many centrality indicators that gauge the importance of a node as an element in a network (Wasserman & Faust, 1994). A well-studied example is betweenness centrality (Anthonisse, 1971; Freeman, 1977), which measures the extent to which shortest paths between nodes in the network pass through a given node. As such, it characterizes this node's control over the information flow through the network.

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http://dx.doi.org/10.1016/j.joi.2013.06.002

Depending on the network, it may happen that each node belongs to a larger subgroup. For instance, authors belong to a department, a university or a country; articles belong to a journal; journals belong to a publisher's portfolio or a scientific discipline. Typically, some nodes are only important within their own group, whereas others are 'brokers' or 'bridges' between several groups in the network. Flom, Friedman, Strauss, and Neaigus (2004) introduced a new indicator, called *Q-measure*, for the brokerage role of nodes between two groups in a connected, undirected, unweighted network. This original definition was later extended to networks with more than two groups, as well as weighted and directed networks (Guns & Rousseau, 2009; Rousseau & Zhang, 2008). In the case of networks with three or more groups, a global as well as a local Q-measure have been introduced (see Section 2). Q-measures, where only shortest paths between nodes from different groups are taken into account, are another variant of betweenness centrality (Brandes, 2008; Flom et al., 2004).

In this article we aim to contribute to a better understanding of Q-measures by studying some of their mathematical properties in unweighted, undirected networks. The remainder of this article is organized as follows. The next section reviews the definitions of betweenness centrality and global and local Q-measures. Section 3 constitutes the main theoretical contribution of this article by introducing external Q-measures and studying the precise relations between betweenness centrality and Q-measures. In particular we present a convex decomposition of the global Q-measure into a local and an external Q-measure. In Section 4 we present a characterization of nodes with a maximum global, local or external Q-measure (i.e., equal to one if normalization is applied). Finally, the last section presents the conclusions.

2. Q-measures and betweenness centrality: definitions

We assume that we have a network $N=(V, E)$, consisting of a set V of nodes or vertices and a set E of links or edges. A shortest path or geodesic between nodes g and h is denoted as $\gamma_{g,h}$. A geodesic between g and h that passes through a ($a \neq g$, $a \neq h$) is denoted as $\gamma_{g,h}(a)$.

Betweenness centrality is a measure characterizing the importance of a given node in establishing short pathways between other nodes (Anthonisse, 1971; Freeman, 1977). Mathematically, betweenness centrality of a node a is expressed as

$$\sum_{g,h \in V} \frac{p_{g,h}(a)}{p_{g,h}} \qquad (1)$$

where $p_{g,h}=|\gamma_{g,h}|$ is the number of geodesics between nodes g and h ($g \neq h$) and $p_{g,h}(a)=|\gamma_{g,h}(a)|$ is the number of geodesics between nodes g and h that pass through a. Normalizing formula (1) leads to a number between 0 and 1. For an undirected network with n nodes this normalized form leads to formula (2):

$$C_B(a) = \frac{2}{(n-1)(n-2)} \sum_{g,h \in V} \frac{p_{g,h}(a)}{p_{g,h}} \qquad (2)$$

Betweenness centrality has become one of the standard centrality measures in social network analysis, along with degree centrality, closeness centrality and eigenvector centrality (Wasserman & Faust, 1994). Many variants of betweenness centrality have since been proposed, such as group betweenness centrality (Everett & Borgatti, 1999) and edge betweenness centrality (Girvan & Newman, 2002).

In this article we focus mainly on the so-called Q-measures, originally introduced by Flom et al. (2004). If the network consists of two groups G (consisting of m nodes) and H (consisting of n nodes), then the Q-measure of node a is defined as:

$$Q(a) = \frac{1}{TP} \sum_{\substack{g \in G \\ h \in H}} \frac{p_{g,h}(a)}{p_{g,h}} \qquad (3)$$

Here, TP denotes the total number of possible pairs of nodes from the two groups, not including a. If $a \in G$, then $TP=(m-1) \cdot n$ and if $a \in H$, then $TP=(n-1) \cdot m$.

Q-measures have subsequently been studied and applied in Chen and Rousseau (2008), Guns and Liu (2010), Guns, Liu, and Mahbuba (2011), Guns and Rousseau (2009), Rousseau (2005), and Rousseau and Zhang (2008). Rousseau and Zhang (2008) introduced Q-measures for networks with directed and weighted links. Guns and Rousseau (2009) expanded the definition to networks with any finite number of groups and showed that in this case one can define both a global and a local variant. An application of the concept of Q-measures is provided by Guns et al. (2011). These authors study a collaboration network of 1129 researchers from different countries, in the fields of bibliometrics, informetrics, webmetrics, and scientometrics during the period 1990–2009.