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Spatial relations between 3D objects: The association between natural language, topology, and metrics ☆

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ABSTRACT

With the proliferation of 3D image data comes the need for advances in automated spatial reasoning. One specific challenge is the need for a practical mapping between spatial reasoning and human cognition, where human cognition is expressed through natural-language terminology. With respect to human understanding, researchers have found that errors about spatial relations typically tend to be metric rather than topological; that is, errors tend to be made with respect to quantitative differences in spatial features. However, topology alone has been found to be insufficient for conveying spatial knowledge in natural-language communication. Based on previous work that has been done to define metrics for two lines and a line and a 2D region in order to facilitate a mapping to natural-language terminology, herein we define metrics appropriate for 3D regions. These metrics extend the notions of previously defined terms such as splitting, closeness, and approximate alongness. The association between this collection of metrics, 3D connectivity relations, and several English-language spatial terms was tested in a human subject study. As spatial queries tend to be in natural language, this study provides preliminary insight into how 3D topological relations and metrics correlate in distinguishing natural-language terms.

1. Introduction

In tandem with increases in pervasive mobile computing and the proliferation of 3D image data comes the need for advances in automated spatial reasoning. One of the particular challenges is the need for a practical mapping between qualitative and quantitative spatial reasoning and human cognition, the latter being expressed principally through natural-language terminology. With respect to human understanding, errors about spatial relations typically tend to be metric rather than topological [1,2]; however, topology alone has been found to be insufficient for conveying spatial knowledge in natural-language communication [3,4]. The consensus is that topology matters while metrics refine [5]. To accommodate natural-language spatial queries, an effective interface between automated spatial reasoning and natural language requires an appropriate blend of natural language, topology, and metrics.

Based on the work that has been done to define metrics for two lines [4] and a line and a 2D region [3] with topological relations in order to facilitate a mapping to natural-language terminology, herein we define metrics appropriate for two 3D regions and the topological connectivity relations used in VRCC-3D+ [6–8]. These metrics extend the notions of what previous authors [3,4,9] have referred to as: *splitting* (i.e., how much is in common between two objects), *closeness* (i.e., how far apart parts are), and *approximate alongness* (i.e., a combination of splitting and closeness). The association between this collection of metrics, 3D connectivity relations, and several

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English-language spatial terms was tested in a human subject study. The results of that study provide preliminary insight into how the 3D topological relations and metrics correlate in distinguishing natural-language terms.

The paper is organized as follows. Section 2 briefly discusses the region connection calculus, VRCC-3D+, and the topological relations pertinent to this study. Section 3 defines the metric relations for splitting, closeness, and approximate alongness, which are similar in concept to those that have been proposed for a line and a 2D region [3], but are significantly redefined to be appropriate for objects in 3D space. Section 4 identifies dependencies between the topological relations and the metrics, as well as intra-relationships within the metrics. Section 5 examines associations between the topological relations, the metrics, and various natural-language terms based on the results of a human subject experiment. Section 6 outlines directions for future work, followed by a summary and conclusions in Section 7.

2. Topological relations

2.1. Mathematical preliminaries

R^3 denotes the three-dimensional space endowed with a distance metric. Here the mathematical notions of *subset, proper subset, equal sets, empty set* (∅), *union, intersection, universal complement,* and *relative complement* are the same as those typically defined in set theory. The notions of *neighborhood, open set, closed set, limit point, boundary, interior, exterior,* and *closure* of sets are as in point-set topology [10]. The interior, boundary, and exterior of any region are disjoint, and their union is the universe.

A set is *connected* if it cannot be represented as the union of disjoint non-empty open sets. For any non-empty bounded set A, we use symbols A^c, A^i, A^b, and A^e to represent the universal complement, interior, boundary, and exterior of a set A, respectively. Two regions A and B are equal if $A^i = B^i$, $A^b = B^b$, and $A^e = B^e$ are true. For our discussion, we assume that every region A is a non-empty, bounded, regular closed, connected set without holes; specifically, A^b is a closed curve in 2D, and a closed surface in 3D.

2.2. Region connection calculi

Much of the foundational research on qualitative spatial reasoning is based on a region connection calculus (RCC) that describes 2D regions (i.e., topological space) by their possible relations to each other. Most notable is the RCC8 model [11] which defines the following eight relations (illustrated in Fig. 1): disconnected (DC), externally connected (EC), partial overlap (PO), equality (EQ), tangential proper part (TPP), non-tangential proper part (NTPP), converse tangential proper part (TPPc), and converse non-tangential proper part (NTPPc). Topological relations in a region connection calculus are typically defined using first-order logic (as in the work of Randell et al. [11]) or using the 9-Intersection model [12] which looks at whether the intersections between the interiors, exteriors, and boundaries of two regions are empty or non-empty.

Whereas a 2D object is in a plane, a 3D object is in space. The simple examples of 3D objects are a pyramid, a cube, a cylinder, and a sphere. A concave pyramid is a complex, simply connected 3D object. Since concave objects can be partitioned into convex objects, for all practical purposes, we work with convex objects. For the rest of this discussion, we will base our analysis on convex objects; in particular, spheres are used in our natural-language human study.

VRCC-3D+ [6–8] is the implementation of a region connection calculus that qualitatively determines the spatial relations between 3D objects, both in terms of connectivity and obscuration. The VRCC-3D+ connectivity relations are named the same as in RCC8; however, the VRCC-3D+ connectivity relations are calculated in 3D rather than 2D. Fifteen obscuration relations also are defined in VRCC-3D+. Considered from a 2D projection, each VRCC-3D+ obscuration relation is a refinement of basic concepts of no obscuration, partial obscuration, and complete obscuration. A composite VRCC-3D+ relation specifies both a connectivity relation and an obscuration relation. Herein our discussion is limited to the VRCC-3D+ connectivity relations, which heretofore will be referred to as topological relations; application of this work to the VRCC-3D+ obscuration relations is beyond the scope of this paper. For a more in-depth discussion of VRCC-3D+, including how it compares to the other RCC models, see [6–8].

3. Metric properties

Metric relations focus on the quantitative differences in spatial features between the two regions or objects being compared; typically, these relations are expressed as scaled (normalized) volumes, areas, distances, lengths, or size differences. Three metric concepts were introduced in

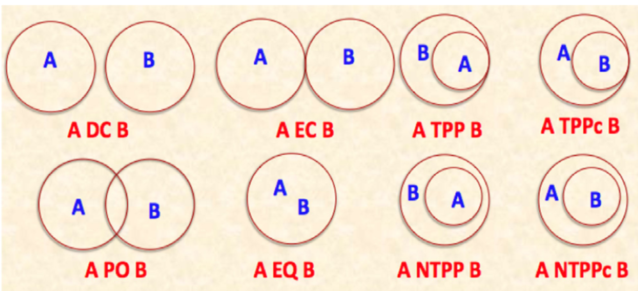

Fig. 1. RCC-8 relations.