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Examples of simple transformations of the *h*-index: Qualitative and quantitative conclusions and consequences for other indices

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Abstract

General results on transformations on information production processes (IPPs), involving transformations of the *h*-index and related indices, are applied in concrete, simple cases: doubling the production per source, doubling the number of sources, doubling the number of sources but halving their production, halving the number of sources but doubling their production (fusion of sources) and, finally, special cases of general power law transformations. In each case we calculate concrete transformation formulae for the *h*-index *h* (transformed into h^*) and we discuss when we have $h^* < h$, $h^* = h$ or $h^* > h$.

These results are then extended to some other *h*-type indices such as the *g*-index, the *R*-index and the weighted *h*-index.

Keywords: *h*-Index; Hirsch; Transformation; *g*-Index; Weighted *h*-index; *R*-Index

1. Introduction

A general information production process (IPP) where sources produce items is characterized by a size-frequency function $f : [a, \rho_m] \to \mathbb{R}^+$ or, equivalently, by a rank-frequency function $g : [0, T] \to \mathbb{R}^+$. Here ρ_m denotes the maximum item density (*a* is the minimum item density) and *T* denotes the total number of sources. For each $j \in [a,\rho_m]$, *f*(*j*) denotes the density of sources with item density j and for each $r \in [0,T]$, $g(r)$ denotes the item density in the source at rank density *r* (see Egghe, 2005 and many papers in the bibliography of Egghe, 2005, e.g. Egghe, 2004).

In Egghe (2007) one studies general transformations of such an IPP: a transformation ψ on the sources:

$$\psi : [0, T] \to [0, T^*] \tag{1}$$

and a transformation φ on the items:

$$\varphi : [a, \rho_m] \to [a^*, \rho_m^*] \tag{2}$$

such that φ, ψ are increasing, $\psi(0) = 0$, $\psi(T) = T^*$, $\varphi(a) = a^*$ and $\varphi(\rho_m) = \rho_m^*$, acting on g as follows: the transformed rank-frequency function g^* satisfies:

$$g^*(r^*) = g^*(\psi(r)) = \varphi(g(r)) \tag{3}$$

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In Egghe (2007) one proves that a Lotkaian size-frequency function:

$$f(j) = \frac{C}{j^\alpha} \tag{4}$$

$C > 0$, $\alpha > 1$, $j \in [1,+\infty[$, is transformed into another Lotkaian size-frequency function:

$$f^*(j^*) = \frac{G}{j^{*\delta}} \tag{5}$$

where

$$\delta = 1 + \frac{b(\alpha - 1)}{c} \tag{6}$$

and $j^* \geq \varphi(a) = Ba^c$, in case φ and ψ are increasing power functions:

$$r^* = \psi(r) = Ar^b \tag{7}$$

$$j^* = \varphi(j) = Bj^c \tag{8}$$

A, B, b and $c > 0$.The power functions (7) and (8) are natural functions to describe evolution of an IPP: they comprise convex and concave growth of sources and items (if the exponents are >1 of <1, respectively). They are also logical to use in connection with the Lotkaian function (4) which is also a power law. Finally, from a pragmatic point of view: only for functions (7) and (8) a simple and exact result as (5), (6) can be proved.

The h-index was defined in Hirsch (2005) in the connection of papers and their citations. In the general IPP context the h-index can be defined as follows (cf. Egghe & Rousseau, 2006a): if we order the sources in decreasing order of their number of items then the h-index of this IPP is the largest rank h such that all the sources on ranks $\leq h$ have at least h items.

The h-index for a general Lotkaian IPP for which (4) is valid, is proved in Egghe and Rousseau (2006a) to be equal to

$$h = T^{1/\alpha} \tag{9}$$

We refer to Ball (2005), Bornmann and Daniel (2005), Braun, Glänzel, and Schubert (2005, 2006), Glänzel (2006), Popov (2005), van Raan (2006) and, of course, to the introductory paper Hirsch (2005) for some background on the advantages and disadvantages of the h-index.Under the above described transformations (7) and (8), we proved in Egghe (in press-a) that the transformed h-index h^* equals:

$$h^* = B^{(\delta-1)/\delta} T^{*(1/\delta)} \tag{10}$$

where B is as in (8) and δ as in (6) (here $T^* = \psi(T)$ is the transformed total number of sources).

It is clear that (10) can be further developed as follows. Since

$$T^* = \psi(T) = AT^b \tag{11}$$

by definition of ψ and by (7), we can put (11) in (10) yielding:

$$h^* = B^{(\delta-1)/\delta} A^{1/\delta} T^{b/\delta} \tag{12}$$

which is the general equation for h^* in Lotkaian systems and where we have power functions (7) and (8) as transformations.

It is proved in Egghe (2007) (but it also follows from (6)) that $\delta = \alpha \Leftrightarrow b = c$. In this case we have

$$h^* = B^{(\alpha-1)/\alpha} A^{1/\alpha} T^{b/\alpha} \tag{13}$$

$$h^* = B^{(\alpha-1)/\alpha} A^{1/\alpha} h^b \tag{14}$$

by (9). Finally, if $b = c = 1$ we have by (14):

$$h^* = B^{(\alpha-1)/\alpha} A^{1/\alpha} h \tag{15}$$