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A survey of Euler diagrams

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ABSTRACT

Euler diagrams visually represent containment, intersection and exclusion using closed curves. They first appeared several hundred years ago, however, there has been a resurgence in Euler diagram research in the twenty-first century. This was initially driven by their use in visual languages, where they can be used to represent logical expressions diagrammatically. This work lead to the requirement to automatically generate Euler diagrams from an abstract description. The ability to generate diagrams has accelerated their use in information visualization, both in the standard case where multiple grouping of data items inside curves is required and in the area-proportional case where the area of curve intersections is important. As a result, examining the usability of Euler diagrams has become an important aspect of this research. Usability has been investigated by empirical studies, but much research has concentrated on *wellformedness*, which concerns how curves and other features of the diagram interrelate. This work has revealed the drawability of Euler diagrams under various wellformedness properties and has developed embedding methods that meet these properties.

Euler diagram research surveyed in this paper includes theoretical results, generation techniques, transformation methods and the development of automated reasoning systems for Euler diagrams. It also overviews application areas and the ways in which Euler diagrams have been extended.

1. Introduction

The embedding, application and theoretical underpinning of Euler diagrams has been the focus of much recent research effort. This paper surveys the state-of-the-art in the area.

Euler diagrams are collections of labelled, closed curves. They visually represent containment, intersection and exclusion. An example is shown in Fig. 1. This diagram can be interpreted as showing that all dogs are mammals, some mammals are pets and some dogs are pets.

The history of Euler diagrams is explored by Baron [4]. She notes that Leibniz produced similar diagrams before Euler, however, much of this work was unpublished. She also observes even earlier Euler-like diagrams by Ramon Lull in the 13th Century. However, logical reasoning using labelled closed curves was first popularized by Leonhard Euler [32] in his ‘Lettres à une Princesse d'Allemagne’. These letters are based on his lessons in physics and philosophy between 1760 and 1762 for Princesse d'Anhalt-Dessau, niece of King Frederick II of Prussia. See Fig. 2 for an example of his drawings.

Alternative names for Euler diagrams include ‘Euler circles’. They can also be incorrectly called Venn diagrams. Venn diagrams require all possible curve intersections to be present, so can be seen as a subset of Euler diagrams, that is, every Venn diagram is an Euler diagram, but not every Euler diagram is a Venn diagram. Venn diagrams were introduced by John Venn a hundred years after Euler [117] and shaded empty regions, a strategy that also appears in some modern Euler diagram syntax. Whilst Venn diagrams quickly become convoluted and difficult to interpret as more curves are added, there are constructions that maintain wellformed properties of the diagrams (wellformedness is discussed in Section 4). There is some evidence to support the use of Euler diagrams

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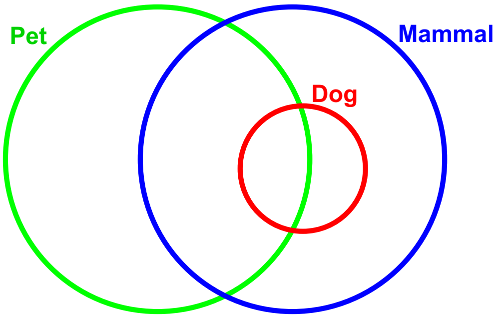

Fig. 1. An example Euler diagram.

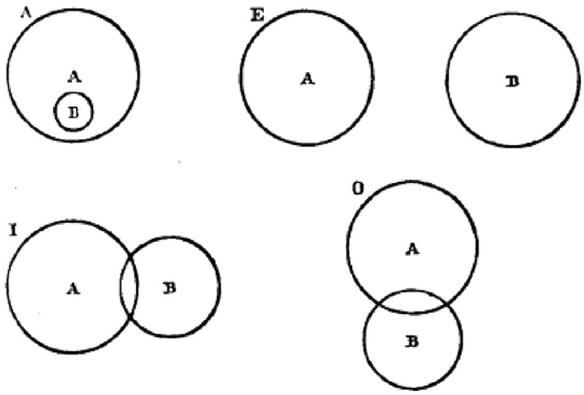

Fig. 2. From Euler's original drawings [32].

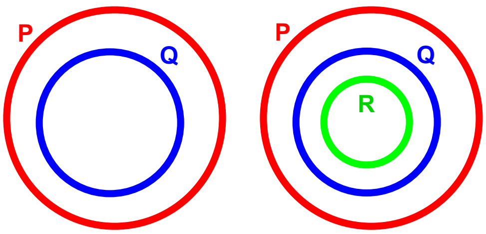

Fig. 3. An illustration of free rides.

over Venn diagrams for deductive reasoning [91]. See Ruskey's excellent survey on Venn diagrams [90] for more details about research in that area.

Euler diagrams are considered to be an effective means of visualizing containment, intersection and exclusion. The use of Euler diagrams as a mechanism to group items is supported by the preattentive processing concept of closure [116]. Euler diagrams are also considered to aid inference, using the notion of a 'free-ride' [5], where adding a curve can allow the deduction of information not present in either the original diagram or added curve. This can be seen in Fig. 3, which is based on an example in [74]. In the leftmost diagram, Q is a subset of P. In the rightmost diagram, R has been added as a subset of Q, which means that R can also be seen to be a subset of P for free. Free-rides, along with the view that many Euler diagram notations are 'well matched to meaning' [49] supports their use in reasoning systems, such as those given in Section 7. Apart from logic and reasoning, Euler diagrams are used in various contexts from Medicine to Software Engineering. Section 2 gives an overview of application areas.

There are alternatives to Euler diagrams. They include LineSets [2], which group items by drawing a line which connects all the items in a set. There is potential for confusion if the user interprets an ordering to the line that is not present or if a line follows a path with many bends. Hypergraphs can also be used for grouping items if drawn in the subset standard [8], however, this method can lead to unwanted, empty, set intersections being present and relies on the notion that items are already laid out, problems that also occur with Bubble Sets [27]. Alternatives for use in logic include Veitch diagrams and Karnaugh maps [13], however these take a rectilinear approach to visualizing intersection, which is not always effective from a usability perspective. Area-proportional Euler diagrams aim to ensure that the regions are of a desired area. This is similar to cartograms [30], where territories are distorted so that their area is of a desired value, representing a quantity associated with the territory (for example, population). However, the construction methods do not consider any closed curves that might surround a number of territories, so using cartograms in place of area-proportional Euler diagrams would mean that the visual containment of a set of regions by a single curve is lost.

The rest of this paper is organized as follows: Section 2 motivates the study of Euler diagrams by exploring the variety of application areas where they are applied; Section 3 gives a definition of Euler diagrams and examines different abstractions and representations of Euler diagrams; Section 4 explores the concept of wellformedness, a crucial component of Euler diagram visualization; Section 5 looks at automated techniques for embedding Euler diagrams; Section 6 provides an outline of systems that transform Euler diagrams; Section 7 gives an overview of the reasoning systems that are based on Euler diagrams; Section 8 examines research in the case where regions are required to be a specified area; Section 9 looks at cases where Euler diagrams have been extended. We end each of these sections by outlining open research questions. Finally, Section 10 concludes.

2. Application areas

The purpose of this section is to give a flavour of the applications that make use of Euler diagrams. It is divided into common application areas. There is no attempt to be complete, the intention is to explore the types of Euler diagram used in the real world to motivate the research discussed in later sections of this paper. It should be noted here that Wilkinson [125] conducted an informal review of articles from the 2009 volumes of Science, Nature, and online affiliated journals and found 72 Venn or Euler diagrams.

2.1. Medical data

The results of medical studies are often visualized as three curve area-proportional diagrams. The goal of such diagrams is to visually communicate scientific results. For example, the three circle Euler diagram from a widely cited study [95] shows the intersections of physician-diagnosed asthma, chronic bronchitis, and emphysema within patients with obstructive lung disease, see Fig. 4. Euler diagrams have become a widely used technique for