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Towards explaining the cognitive efficacy of Euler diagrams in syllogistic reasoning: A relational perspective ☆, ☆☆

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ABSTRACT

Although diagrams have been widely used as methods for introducing students to elementary logical reasoning, it is still open to debate in cognitive psychology whether logic diagrams can aid untrained people to successfully conduct deductive reasoning. In our previous work, some empirical evidence was provided for the effectiveness of Euler diagrams in the process of solving categorical syllogisms. In this paper, we discuss the question of why Euler diagrams have such inferential efficacy in the light of a logical and proof-theoretical analysis of categorical syllogisms and diagrammatic reasoning. As a step towards an explanatory theory of reasoning with Euler diagrams, we argue that the effectiveness of Euler diagrams in supporting syllogistic reasoning derives from the fact that they are effective ways of representing and reasoning about relational structures that are implicit in categorical sentences. A special attention is paid to how Euler diagrams can facilitate the task of checking the invalidity of an inference, a task that is known to be particularly difficult for untrained reasoners. The distinctive features of our conception of diagrammatic reasoning are made clear by comparing it with the model-theoretic conception of ordinary reasoning developed in the mental model theory.

1. Introduction

In logic teaching, Venn and Euler diagrams have been widely used as tools for introducing students to elementary logical reasoning, including set-theoretical and syllogistic reasoning. However, in the literature of the cognitive psychology of reasoning, it is still open to debate whether external diagrams can aid logically untrained people to conduct deductive reasoning in a successful way (see [51] for an overview of the study of external representations). Indeed, it is often claimed that diagrams can only serve as an auxiliary source of information in deductive problem solving. Thus, Larkin and Simon [28], in a seminal work on the efficacy of diagrammatic representations in problem solving in general, argued that reasoning is largely independent of ways of representing information, and hence, that diagrams are less beneficial in reasoning than in such tasks as searching and recognition. Additionally, previous studies reported empirical evidence for negative effects of traditional Euler diagrams on the performance of syllogistic reasoning (Calvillo et al. [7]; Rizzo and Palmonari [45]). Furthermore, various systems of logic diagrams have been proposed and studied using the methods of mathematical logic, such as Venn–Peirce diagrams [52], Euler diagrams [21], Constraint diagrams [13,27], Spider diagrams [24], and Concept diagrams [9]; see Stapleton [56] and Howse [23] for surveys. However, there are few empirical studies on how effective such diagrammatic systems are in people's actual reasoning (but see Section 2 for related work).

In view of this situation, we have studied how logic diagrams can support actual deductive reasoning, focusing

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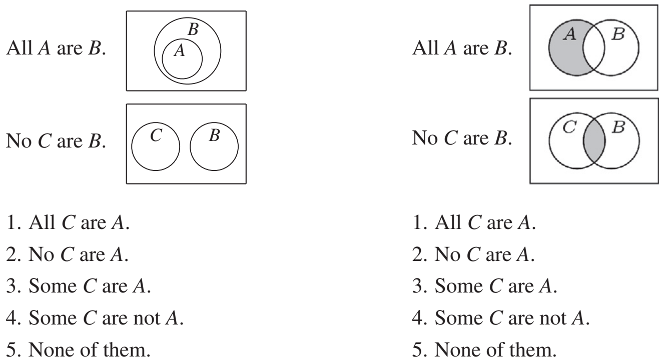

Fig. 1. Examples of syllogistic reasoning tasks with Euler diagrams (left) and Venn diagrams (right). The correct answer is "No *C* are *A*."

on the case of syllogistic reasoning supported by Euler and Venn diagrams that are externally given to reasoners [32,47,48]. Typical examples of reasoning tasks that we examined are shown in Fig. 1.

Euler diagrams represent set relationships in terms of inclusion and exclusion relations between circles (or, more generally, simple closed curves); see the left side diagrams in Fig. 1. By contrast, Venn diagrams have a fixed configuration of circles and represent set relationships by stipulating that shaded regions denote the empty set; see the right side diagrams in Fig. 1. In the experiments of [47,48], subjects were divided into three groups, called the Euler group, Venn group, and Linguistic group. The Euler group and Venn group were provided with instructions on the meanings of diagrams. A pretest was conducted to check whether the subjects understood the instructions correctly. The Euler group was then asked to solve syllogistic reasoning tasks in which subjects were presented with two sentential premises together with two corresponding Euler diagrams, as in the left side of Fig. 1, and asked to choose a valid conclusion from five possibilities. Similarly, the Venn group was asked to solve tasks as in the right side of Fig. 1. The Linguistic group was presented only with sentential premises and required to choose a valid conclusion without any aid from diagrams. The results showed that (1) the performance of the Euler and Venn groups was significantly better than that of the Linguistic group, and that (2) the performance of the Euler group was significantly better than that of the Venn group.

It should be noted that in the experimental setup of [47,48], subjects in the Euler and Venn groups were given instructions on the meaning of diagrams, while subjects in the Linguistic group were not. Then one might argue that the difference in training could have had a major effect on differences in performance between the Euler and Venn groups, on the one hand, and the Linguistic group, on the other. However, such an objection can be avoided if a comparison is made between the Euler group and the Venn group. The latter was also given substantial instructions and practice trials, yet the result showed that the performance of the Euler group was significantly better than that of the Venn group.

The aim of the present paper is to discuss these experimental results in the light of the formal analyses of reasoning with categorical sentences and reasoning with Euler diagrams, presented in [35,36], respectively. More specifically, the central aim is to defend and motivate the following hypothesis: the effectiveness of Euler diagrams in supporting syllogistic reasoning derives from the fact that they are effective ways of representing and reasoning about *relational* structures that are implicit in categorical (quantified) sentences. In claiming this, we are trying to make a connection between the two lines of research, namely, experimental studies of syllogistic reasoning supported with Euler diagrams [47,48] and logical (proof-theoretical) studies of syllogistic and diagrammatic reasoning [35,36]. Before the current paper and its workshop version [37], these two lines of research were unconnected; thus, neither the hypotheses put forward in the experimental papers [47,48], nor the discussion section of these papers, mentioned the relational analysis of syllogistic reasoning as presented in [35,36]. And also, our study in [35,36] is concerned with purely formal aspects of syllogistic and diagrammatic reasoning and hence does not discuss its application to cognitive experimental studies of diagrammatic reasoning. The present paper is the first substantial attempt to bridge the logical and cognitive studies of Euler diagrams that we have developed in recent years.

The formal study of logic diagrams in [35] also sheds light on the question of how diagrams can contribute to judging that a given inference is *invalid* in actual reasoning. It has been noticed in cognitive psychology of reasoning that falsification tasks, including tasks that require a reasoner to judge that there is no valid conclusion drawable from a given set of premises, are often difficult for untrained people when inference materials are only presented in linguistic (sentential) form. Interestingly, the experimental results in [47] showed that Euler diagrams were particularly effective in supporting such falsification tasks of syllogistic reasoning. We will argue that the efficacy of Euler diagrams in falsification tasks is partly explained by assuming that when such diagrams are externally given, the information that there is no valid conclusion drawable from the premise diagrams can be obtained in a direct way, specifically, by combining premise diagrams and extracting the relevant relational information. This way of understanding diagrammatic reasoning can be made clear by comparing it with model-based inferences such as those studied in the mental model