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A diagrammatic reasoning system for the description logic $\mathcal{ALC}$

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Abstract

Diagrammatic reasoning is a tradition of visual logic that allows sentences that are equivalent to first order logic to be written in a visual or structural form: usually for improved usability. A calculus for the diagram can then be defined that allows well-formed formulas to be derived. This calculus is intended in the analog of logical inference.

Description logics (DLs) have become a popular knowledge representation and processing language. DLs correspond to decidable fragments of first order logic; their notation is in the style of symbolic, variable-free formulas. Moreover, DLs are equipped with table au theorem provers that are proven to be sound and complete.

Although DLs have roots in diagrammatic languages (such as semantic networks), they are elaborated in a purely symbolic manner. This paper discusses how DLs can be equivalently represented in terms of a diagrammatic reasoning system.

First, existing diagrammatic reasoning systems, namely spider- and constraint diagrams, as well as existential and conceptual graphs, are investigated to determine if they are compatible with DLs. It turns out that Peirce's existential graphs are better suited for this purpose than the alternatives we examine.

The paper then redevelops the DL $\mathcal{ALC}$, which is the smallest propositional DL, by means of labeled trees, and provides a diagrammatic representation for these trees in the style of Peircean graphs. We provide a calculus based on C.S. Peirce's calculus for existential graphs and prove the soundness and completeness of the calculus. The calculus acts on labeled trees, but can be best understood as a diagrammatic calculus whose rules modify the Peircean-style representation of $\mathcal{ALC}$.

Keywords: Description logics; Spider diagrams; Constraint diagrams; Conceptual graphs; Existential graphs; Logic; Syntax; Semantics; Calculus

1. Introduction

Description logics (DLs) are a common family of knowledge representation languages tailored to express knowledge about concepts and concept hierarchies. They include sound and complete decision procedures for reasoning about such knowledge. One of the main applications of DLs is their use as the basis for an ontology

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language, especially popular for the Semantic Web. In particular, the Ontology Web Language (OWL)—a W3C recommendation for the knowledge language of the Semantic Web—is based on a specific and expressive DL termed $\mathcal{SHOIN}(D)$.[1]

The basic building blocks of DLs are concepts (unary predicates), roles (binary relations) and sometimes individuals, which can be composed by language constructs such as intersection, union, value or number restrictions to build more complex well-formed formulas that themselves represent complex concepts and roles. For example, when MAN, FEMALE, MALE, RICH, HAPPY are predefined concepts and if HASCHILD is a predefined role, then

$$\text{MAN} \sqcap \exists\text{HASCHILD.FEMALE} \sqcap \exists\text{HASCHILD.MALE} \sqcap \forall\text{HASCHILD.}(\text{RICH} \sqcup \text{HAPPY}) \quad (1)$$

describes the concepts of men who have both male and female children, and where all the children are rich or happy. Let us call a concept defined in this way as HAPPYMAN.

The formal notation of DLs has the flavour of a variable-free first order predicate logic (FOL). In fact, DLs correspond to (decidable) fragments of FOL, and like FOL, DLs have a well-defined, formal syntax, a semantics in the form of Tarski-style models, and a sound and complete calculi (based on Table aux-algorithms). It is often emphasised that DLs offer, in contrast to other knowledge representation languages, sound, complete and (empirically) tractable reasoning services. A comprehensive overview on DLs is given in the *Description Logic Handbook* [2].

The notation of DLs is in the style of the usual linear and symbolic[2] notations of FOL. The fact that the notation of DLs is variable-free makes them easier to comprehend than the common FOL formulas which include free variables. Nonetheless, for untrained users, the symbolic notation of DLs can be hard to learn and comprehend.

A main alternative to the symbolic notation is the development of a diagrammatic representation of DLs. It is well accepted that diagrams are in many cases easier to comprehend than symbolic notations (see for example [3–6]), and in particular it has been argued that they are useful for knowledge representation systems [7,8]. This has been acknowledged by the DL community and is a common view among the broader knowledge representation community [9]. In [10], the introduction to the *Description Logic Handbook*, Nardi and Brachman write that besides the possibility of "providing a syntax that resembles more closely natural language", a "major alternative for increasing the usability of DL as a modeling language" is to "implement interfaces where the user can specify the representation structures through graphical operations."

A first attempt at a diagrammatic representation for DL is can be found in [7], where Gaines elaborates a graph-based representation for the textual DL CLASSIC, part of the KL-ONE-framework. More recently, the focus has shifted from the development of proprietary diagrammatic representations to representations within the framework of UML (unified modeling language). In 2003, the *Object Management Group* requested a metamodel for the purpose of defining ontologies. Following this proposal, Brockmans et al. [11] provide a UML-based, diagrammatic representation for the OWL DL. In these approaches, the focus is on a graphical *representation* of DL, however, as emphasized in many works on DL (see for example [2]), *reasoning* is seen as a distinguishing feature of DL and such reasoning is not supported diagrammatically by that treatment. Correspondences between graphical representation of the DL and the DL reasoning system are therefore important but remain largely unelaborated to date. Similar arguments hold for other popular diagrammatic languages like UML and ORM[3] the difference being that, that unlike DLs, these diagrammatic modeling languages provide no extensional, mathematical semantics, nor any automated reasoning facilities.

On the other hand, there are some candidate diagrammatic reasoning systems that have the expressiveness of fragments of FOL, or even full FOL. In this paper, we will evaluate two families of

[1]In $\mathcal{SHOIN}(D)$, $\mathcal{S}$ stands for the basic DL $\mathcal{ALC}$ (equivalent to the propositional modal logic extended with transitive roles), $\mathcal{H}$ stands for role hierarchies, $\mathcal{O}$ stands for nominals (classes whose extension is a single individual), $\mathcal{N}$ stands for unqualified number restrictions and D stands for datatypes) [1].

[2]We use the term 'symbolic' according to C.S. Peirce's classification of signs into 'icons', 'indices' and 'symbols'. See Shin [3] for an introduction.

[3]http://www.orm.net/index.html