Transportation Research Part C 69 (2016) 276–290

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Global convergence of the trial-and-error method for the traffic-restraint congestion-pricing scheme with day-to-day flow dynamics

Min Xu [a], Qiang Meng [a,*], Zhongxiang Huang [b]

[a] Department of Civil and Environmental Engineering, National University of Singapore, Singapore 117576, Singapore
[b] School of Traffic and Transportation Engineering, Changsha University of Science & Technology, Changsha, Hunan 410004, PR China

ARTICLE INFO

Article history:
Received 17 December 2015
Received in revised form 8 June 2016
Accepted 8 June 2016
Available online 17 June 2016

Keywords:
Trial-and-error method
Traffic-restraint congestion-pricing scheme
Day-to-day flow dynamics
Global convergence

ABSTRACT

The traffic-restraint congestion-pricing scheme (TRCPS) aims to maintain traffic flow within a desirable threshold for some target links by levying the appropriate link tolls. In this study, we propose a trial-and-error method using observed link flows to implement the TRCPS with the day-to-day flow dynamics. Without resorting to the origin–destination (O–D) demand functions, link travel time functions and value of time (VOT), the proposed trial-and-error method works as follows: tolls for the traffic-restraint links are first implemented each time (trial) and they are subsequently updated using observed link flows in a disequilibrium state at any arbitrary time interval. The trial-and-error method has the practical significance because it is necessary only to observe traffic flows on those tolled links and it does not require to wait for the network flow pattern achieving the user equilibrium (UE) state. The global convergence of the trial-and-error method is rigorously demonstrated under mild conditions. We theoretically show the viability of the proposed trial-and-error method, and numerical experiments are conducted to evaluate its performance. The result of this study, without doubt, enhances the confidence of practitioners to adopt this method.

1. Introduction

Congestion-pricing has been recognized as an effective means of mitigating traffic congestion and has received considerable attention in literature on the subject over the past few decades. The first-best and second-best congestion-pricing principles are fundamental in determining an appropriate congestion toll solution to help achieve a system optimum. As a special second-best congestion-pricing scheme, the traffic-restraint congestion-pricing scheme (TRCPS) aims to maintain traffic flow within a desirable threshold (e.g., the environmental capacity) for some target links or to eliminate traffic queues for the bottleneck links by levying the appropriate link-based tolls (Jones and Hervik, 1992; Yang and Bell, 1997). The existing models and solution algorithms for analytically determining the first-best or second-best congestion-pricing solution, including the traffic-restraint congestion-pricing solution, require the following inputs: explicit and analytical mathematical expressions of origin–destination (O–D) demand functions and link travel time functions, and the specific value of time (VOT) of drivers (Button and Verhoef, 1998; Evans, 1992; Rothengatter, 2003; Verhoef et al., 1996; Verhoef, 2002; Vickrey, 1969; Yang and Huang, 1998). These inputs, however, can hardly be obtained precisely or easily in practice, especially the O–D demand

* Corresponding author.
E-mail address: ceemq@nus.edu.sg (Q. Meng).

http://dx.doi.org/10.1016/j.trc.2016.06.009
0968-090X/

functions and the VOT. This inevitably hinders the adoption of those congestion-pricing studies by practitioners. Without resorting to the O–D demand functions, the trial-and-error methods using observed traffic flows to determine these congestion-pricing solutions have recently been proposed and examined, and will be reviewed in the next subsection. These methods use the generic trial-and-error procedure proposed by Vickrey (1993) and Downs (1993) as follows: a trial using a set of arbitrary tolls is implemented and the resultant traffic flows are subsequently observed, a new set of tolls is then set using the observed traffic flows and implemented for the next trial, and this iteration is repeated until a desirable toll solution is achieved.

The convergence of the existing trial-and-error methods with observed equilibrium traffic flows for the first-best and second-best congestion-pricing solutions relies on the fundamental assumption that the user equilibrium (UE) or stochastic user equilibrium (SUE) traffic flow pattern should be achieved before the next trial of congestion tolls. In other words, we have to wait for a long time before a network flow pattern goes to an equilibrium state each time the newly updated link tolls are implemented. It is also challenging to assess whether a network flow pattern is in the equilibrium state in practice. In reality, once a new congestion-pricing solution is implemented, network users take time to learn the traffic conditions of a network and their route choices are adjusted day by day. Consequently, the resultant link or path flow pattern will evolve over a long period of time before reaching the equilibrium state (UE or SUE). As a matter of fact, the global convergence of the trial-and-error methods proposed by the existing studies may not be guaranteed if the congestion tolls are updated using the observed disequilibrium traffic flows each time. It is therefore very important to investigate the global convergence of the trial-and-error methods designed for the first-best and second-best congestion-pricing scheme, especially the TRCPS by taking into account the day-to-day flow dynamics, in which observed disequilibrium link flows are used to update the interim congestion tolls. This study aims to design a trial-and-error method for the TRCPS with day-to-day flow dynamics and rigorously demonstrate the global convergence of the designed trial-and-error method.

1.1. Literature review

1.1.1. Trial-and-error methods with observed equilibrium traffic flows

For the first-best congestion-pricing scheme, Li (2002) was the first one to propose a bisection method in the absence of the O–D demand functions to realize the trial-and-error procedure with the observed UE traffic flow for a single road. Yang et al. (2004) extended this to the general networks by means of the method of successive averages, and it was subsequently investigated by Zhao and Kockelman (2004, 2006) under the logit-based SUE conditions. The trial-and-error method proposed by Yang et al. (2004) was further improved by Han and Yang (2009) in the convergence rate, and was generalized by Xu et al. (2013) to transportation networks with multiple vehicle types and multiple time periods. Moreover, Wang and Yang (2012) demonstrated the non-convergence of the bisection method proposed by Li (2002) and explored the implementation of the trial-and-error method in the context of tradable credit schemes to achieve a system-optimal state on a single road, and later this method was extended to general networks (Wang et al., 2014). With regard to the second-best congestion-pricing scheme, Yang et al. (2005) proposed another interesting trial-and-error method under the UE conditions and its global convergence remains an unanswered question. It should be pointed out that these trial-and-error methods require the explicit mathematical expression of link travel time functions and the specific VOT.

With regard to the TRCPS, without the aid of the O–D demand functions, link travel functions or the VOT, Meng et al. (2005) suggested another type of trial-and-error method with global convergence under the UE conditions. This method was extended by Meng and Liu (2011) for the logit-based SUE conditions and by Yang et al. (2010) for the asymmetric link travel time functions. Zhou et al. (2015) proposed another trial-and-error method in the absence of the O–D demand functions to achieve the first-best congestion-pricing solution subject to the link capacity constraints.

1.1.2. Day-to-day congestion pricing and trial-and-error methods with day-to-day flow dynamics

To describe the response of network users to exogenous traffic factors, e.g., congestion pricing, the day-to-day flow dynamics formulate the disequilibrium trajectory of the network users' adjustment process before achieving the UE or SUE state (Cantarella, 2013; Friesz et al., 1994; Guo et al., 2013; Han and Du, 2012; Horowitz, 1984; Watling, 1999). Most studies on the day-to-day flow dynamics have examined the invariance principle, the uniqueness of the stationary point and the stability and convergence of the equilibrium state (Bie and Lo, 2010; Smith, 1984).

The early studies mostly focused on the path-based flow dynamics established on the continuous temporal dimension by using a wide range of behavior rules and explicit equations for the evolutionary trajectory of path flows. These studies included the five continuous day-to-day path flow dynamics: the simplex gravity flow dynamics (Smith, 1983), the proportional-switch adjustment process (Smith, 1984), the network tatonnement process (Friesz et al., 1994), the projected dynamical system (Nagurney and Zhang, 2012), and evolutionary traffic dynamics (Sandholm, 2002). Zhang et al. (2001) created a few examples to highlight the importance of appropriate route choice assumptions to ensure the equivalence between the UE solution and the stationary link/path flow pattern from day-to-day flow dynamics. They also postulated that the aggregate cost for the entire network based on the previous day's path costs should decrease day by day until a stationary link/path flow pattern is reached. They further demonstrated that this fundamental assumption can guarantee equivalence between the UE solution and the stationary link/path flow pattern of day-to-day flow dynamics in a network with fixed demand. Yang and Zhang (2009) defined the continuous adjustment process of path flows for a network with fixed demand under the assumption of the rational behavior adjustment process (RBAP). The RBAP was proven to be a general form of the