Sustainable Energy, Grids and Networks 5 (2016) 27–38

Contents lists available at ScienceDirect

Sustainable Energy, Grids and Networks

journal homepage: www.elsevier.com/locate/segan

# Real-time active and reactive power regulation in power systems with tap-changing transformers and controllable loads☆

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ARTICLE INFO

*Article history:*
Received 20 November 2014
Received in revised form
28 August 2015
Accepted 26 October 2015
Available online 4 November 2015

*Keywords:*
Power system
Optimization
Distributed control
Controllable load
Tap-changing transformer

ABSTRACT

Due to increasing uncertainties resulting from renewable energy penetration and variability in both supply and demand, the control and economic optimization for power networks will need to run on faster time-scales. Moreover, distributed and decentralized control architectures are necessary as power systems are distributed large-scale networks with a lot of complexity, which makes centralized control expensive, inefficient and hard to implement. In this paper, the problem of real-time active and reactive power regulation for power networks with controllable loads and tap-changing transformers is considered. First, the state-space model of a transmission level network under exogenous disturbances is obtained, which is decomposed into two subsystems, describing frequency and voltage dynamics respectively. For the subsystem describing frequency dynamics, an optimization problem relating to active power regulation is first formulated. A distributed controller is then proposed and its optimality, stability and delay robustness are studied. Also, nonlinear proportional actions for controllable loads are introduced to improve the performance of the subsystem. For the subsystem describing voltage dynamics, simple integral control is used for controllable loads and the asymptotic stability of the closed-loop subsystem is shown. To implement the reactive power regulation scheme, a method for local users to detect the tap position variation of the On-Load Tap Changer (OLTC) at the bus they are connected to is presented. Thus, the overall load control scheme is completely decentralized and can be applied easily. Numerical investigations illustrate the performance of the combined active and reactive power regulation scheme.



## 1. Introduction

The operation and control of the modern power grid is layered. Each layer is characterized by its operational/control objectives, corresponding to one time-scale. When looking at one of the layers, the influence from other layers can be ignored because of this architecture. For example, if considering long term contracts (slow time-scale) between a generation company and a factory, the operation and control in the transmission network (fast time-scale) is neglected. This separation works well when the time-scales of the two layers are significantly different. Even if the difference is less obvious, e.g., between seconds and minutes, it is still acceptable because of high predictability and low uncertainty in the system [1]. However, as renewable energy, distributed generation and dynamic demand are introduced, the power grid will face increasing uncertainty and variability in both supply and demand, especially in the real-time layer (time-scale of usually 5 min). In order to maintain system-wide efficiency, reliability and robustness under these changes, redesigning conventional controls and actions and merging different layers/time-scales together becomes more and more necessary. On the other hand, power systems are large-scale and complicated. Because of the increase in uncertainty and variability, conventional centralized control for power networks becomes expensive and less efficient. Therefore, distributed and decentralized control frameworks are necessary, not only to deal with the problem of *scale*, but also to reduce the requirement for communication which can affect system control and optimization in real-time.

☆ Some of the material in this paper appeared in the proceedings of the 53rd IEEE Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA. The work of X. Zhang was supported by the China Scholarship Council (CSC). The work of A. Papachristodoulou was supported in part by the Engineering and Physical Sciences Research Council projects EP/J012041/1, EP/J010537/1 and EP/M002454/1. R. Kang and M. McCulloch would like to thank Martin Lee from Southern and Scottish Energy Power Distribution for providing the tap change data and advice on the detection algorithms.

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http://dx.doi.org/10.1016/j.segan.2015.10.006
2352-4677/

The redesign of conventional control approaches and actions that combines different layers/time-scales together, in either distributed, decentralized or centralized manners is an area of active research. For example, [1] presented a novel control scheme for achieving optimal power balancing and congestion management in power systems via nodal prices, and introduced the concept of autonomous power networks. In [2,3], a distributed real-time control architecture was proposed which merged primary, secondary, and tertiary control for power networks, so as to achieve system-wide efficiency and reliability. In [4], the authors proposed both a decentralized controller and a centralized controller for frequency regulation and generation cost minimization in power systems. In [5], conventional Automatic Generation Control (AGC) was combined with Economic Dispatch (ED) automatically and dynamically, providing an optimization viewpoint to modify AGC. In [6], the authors designed a distributed real-time frequency control scheme through reverse- and forward-engineering. In [7], a hierarchical transactive control architecture was presented which combined market transactions at three different time-scales. In [8], decentralized and distributed primary, secondary, and tertiary control strategies in microgrids were studied, showing that the adoption of droop control was necessary and sufficient to achieve economic optimization in microgrids. In [9], a fast-acting decentralized load control scheme was presented for primary frequency regulation in power networks. In [10], a frequency preserving optimal load control scheme was proposed, which was used to balance power and restore the nominal frequency after a disturbance.

In this paper, conventional primary and secondary control for power systems with tap-changing transformers [11] are merged and redesigned. Moreover, the participation of controllable loads [12] in both active and reactive power regulation is considered, which is unlike most previous work. As shown in [12,13], the use of controllable loads can provide significant added stability and robustness to power systems, especially under real-time disturbances, contingency and renewable energy penetration. The contribution of this paper includes: (i) a distributed generation control scheme which asymptotically stabilizes the frequency dynamics regardless of communication delay and minimizes aggregate power generation cost; (ii) a decentralized load control scheme which consists of proportional actions in real power regulation and integral actions in reactive power regulation, and is simple for realization; and (iii) a method for determining tap change events which can be used to estimate the tap position variation of On-Load Tap Changers (OLTCs), thus, assisting decision making for the load control scheme.

The rest of the paper is organized as follows: In Section 2, the complete state-space model of a transmission level network under exogenous disturbances is presented, where the system is decomposed into two subsystems, i.e., frequency dynamics and voltage dynamics. In Section 3, a distributed generation control scheme and a decentralized load control scheme are proposed to regulate active power for the subsystem describing frequency dynamics, and the optimality, stability and delay robustness of the closed-loop subsystem are shown. In Section 4, a decentralized reactive power regulation scheme for the subsystem describing voltage dynamics is proposed. In order to implement this (reactive power control) scheme, a method for detecting tap position variation of OLTCs at the end users is proposed. In Section 5, numerical examples are presented to illustrate the properties of the control scheme. Conclusions and future work are presented in Section 6.

### 1.1. Notation

$|\mathcal{X}|$ is the cardinality of a set $\mathcal{X}$. $x^0$ denotes the nominal value of a state variable $x$. $x^T$ denotes the transpose of $x$. diag$\{\star\}$ denotes a diagonal matrix with corresponding entries $\star$ on the main diagonal. $X \in \mathbb{R}^{m\times n}$ or $X = [\star]_{m\times n}$ is an $m \times n$ real matrix. $\mathbf{1}_{m\times n}$ ($\mathbf{0}_{m\times n}$) denotes an $m$ by $n$ matrix whose entries are 1 (0) and $I_m$ denotes an identity matrix of size $m \times m$. $x^*$ denotes the equilibrium of a state variable $x$. tr$(X)$ is the trace of a square matrix $X$. $X \succeq 0 (X \succ 0)$ denotes that a square matrix $X$ is positive semi-definite (positive definite). $\mathcal{C}_{n,\tau}$ denotes the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence, where $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. The norm on $\mathcal{C}_{n,\tau}$ is defined as $\|\phi\|_{\mathcal{C}} = \sup_{-\tau\le\theta\le 0} |\phi(\theta)|$ where $|\star|$ is the Euclidean norm for vectors and the induced matrix norm for matrices. Finally, the projection of a function $h(y)$ on a variable $x \in [\underline{x}, \overline{x}]$ is denoted by $(h(y))_{\underline{x}}^{\overline{x}}$ where

$$(h(y))_{\underline{x}}^{\overline{x}} = \begin{cases} \min\{0, h(y)\} & \text{if } x = \overline{x} \\ h(y) & \text{if } \underline{x} < x < \overline{x} \\ \max\{0, h(y)\} & \text{if } x = \underline{x}. \end{cases}$$

## 2. System model

### 2.1. Network model

Consider a transmission (subtransmission) level network with arbitrary topology, described by a connected directed graph $(\mathcal{G} \bigcup \mathcal{L}, \mathcal{E})$. Here $\mathcal{G}$ is the set of generator buses, $\mathcal{L}$ is the set of load buses, and $\mathcal{E} \subseteq (\mathcal{G} \bigcup \mathcal{L}) \times (\mathcal{G} \bigcup \mathcal{L})$ is the set of transmission lines connecting the buses. Each generator bus contains only one synchronous generator/generation unit, and each load bus contains only one load which is an aggregation of a certain amount of users at the bus it is connected to (alternatively speaking, each load corresponds to a distribution level network). This can be realized by introducing fictitious buses [14]. Moreover, all load buses are equipped with transformers containing OLTCs. Number the generator buses $1, \ldots, m$ ($\mathcal{G} = \{1, \ldots, m\}$), the load buses $m+1, \ldots, m+n$ ($\mathcal{L} = \{m+1, \ldots, m+n\}$), and the transmission lines $1, \ldots, p$ corresponding to a lexicographic ordering where $p = |\mathcal{E}|$. Define an orientation from bus $i$ to bus $j$ if $(i, j) \in \mathcal{E}$, $i < j$. View all buses as voltage sources. Denote the voltage of each generator bus by $v_i\angle\delta_i$, $i \in \mathcal{G}$, and the primary-side voltage of the OLTC at each load bus by $v_i\angle\delta_i$, $i \in \mathcal{L}$. Assume the network is working around a nominal operating point which is determined by an ED problem at a more slower time-scale. To simplify the notation, in the rest of the paper, all state variables denote *time-varying deviations* from their nominal operating values, and moreover, all state variables with *tilde* denote their nominal operating values plus their time-varying deviations (from the nominal operating values), i.e., $\tilde{x} = x^0 + x$ where $x$ is the time-varying deviation ($x^*$ denotes the equilibrium of $x$).

**Generator bus model**. First, the following assumption holds for the generator bus model.

**Assumption 1.** Generator bus voltage magnitudes are fixed, i.e., $v_i = 0$, $\forall i \in \mathcal{G}$ (alternatively, $\tilde{v}_i = v_i^0$).

This is reasonable since generator bus voltage magnitudes can be well and fast controlled through excitation systems/Automatic Voltage Regulators (AVRs) [15] when the system is operating around a nominal state. Similar assumptions are also used in [16–18]. Let $\omega_i = \dot{\delta}_i$ be the time-varying frequency deviation from the nominal frequency $\omega_i^0$, e.g., $2\pi \times 50$ Hz in Europe, at bus $i$, $i \in \mathcal{G} \bigcup \mathcal{L}$. The dynamics of generator buses are given by the (linearized) swing equations [14]

$$M_i\dot{\omega}_i + D_i\omega_i = P_{M_i} - P_i - \sum_{(i,j)\in\mathcal{E}} P_{ij}, \quad i \in \mathcal{G} \tag{1}$$