Transportation Research Part C 44 (2014) 318–349

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

High-resolution numerical relaxation approximations to second-order macroscopic traffic flow models

A.I. Delis *, I.K. Nikolos, M. Papageorgiou

School of Production Engineering & Management, Technical University of Crete, Chania, Greece

ARTICLE INFO

Article history:
Received 23 January 2014
Received in revised form 8 April 2014
Accepted 10 April 2014

Keywords:
Second-order traffic flow models
Numerical modeling
Relaxation schemes
High-resolution

ABSTRACT

A novel numerical approach for the approximation of several, widely applied, macroscopic traffic flow models is presented. A relaxation-type approximation of second-order non-equilibrium models, written in conservation or balance law form, is considered. Using the relaxation approximation, the nonlinear equations are transformed to a semi-linear diagonilizable problem with linear characteristic variables and stiff source terms. To discretize the resulting relaxation system, low- and high-resolution reconstructions in space and implicit–explicit Runge–Kutta time integration schemes are considered. The family of spatial discretizations includes a second-order MUSCL scheme and a fifth-order WENO scheme, and a detailed formulation of the scheme is presented. Emphasis is given on the WENO scheme and its performance for solving the different traffic models. To demonstrate the effectiveness of the proposed approach, extensive numerical tests are performed for the different models. The computations reported here demonstrate the simplicity and versatility of relaxation schemes as solvers for macroscopic traffic flow models.

1. Introduction

Traffic flow modeling has attracted a rapidly growing interest over the past years, due to the need for optimization of the usage of existing traffic infrastructure and the design of new structures, the growing economic/environmental cost of traffic jams, their impact on the quality of life, the need to assess and optimize new driver assistance devices, and (from a scientific point of view) the existence of complicated non-linear dynamical phenomena associated with traffic flow and the characteristics of vehicles and drivers' behavior. The effective modeling of such phenomena, like traffic jams, stop-and-go traffic and "synchronized" congested traffic (Helbing et al., 2002), call for the development of accurate theoretical models which should reproduce, at least qualitatively, all traffic states with the minimum number of parameters (Helbing, 2001) and the corresponding computational procedures, which should be numerically efficient (in terms of computational time and memory requirements), accurate and robust, to allow for their use in real-world simulation and optimization scenarios.

Several approaches have been followed over the past years to model the dynamics of traffic flow, the most successful being cellular automata, microscopic car-following models, gas-kinetic models, and macroscopic traffic models. Comprehensive descriptions of such models can be found, for example, in Helbing (2001); Hoogendoorn and Bovy (2001); Bellomo and Dogbe (2011); Treiber and Kesting (2013). Macroscopic models consider the traffic as an anisotropic fluid (continuum), where vehicle dynamics are described in terms of spatial vehicle density and average velocity, as functions of location and time. Some of the advantages of the macroscopic models are (a) their numerical efficiency (compared to microscopic

* Corresponding author. Tel.: +30 2821037751.
E-mail address: adelis@science.tuc.gr (A.I. Delis).

http://dx.doi.org/10.1016/j.trc.2014.04.004
0968-090X/

ones), (b) their good agreement with empirical data, (c) their suitability for analytical investigations, (d) the simple treatment of inflows, and (e) the possibility to simulate multi-lane flows by effective one-lane models, we refer, for example, to Helbing (2001) and Helbing et al. (2002) for thorough discussions.

The main motivation behind this work is the development of a common numerical framework for the macroscopic simulation of traffic flow networks, transparent to the traffic flow model used (only second-order non-equilibrium traffic flow models are considered in this work). New models, especially for Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) simulations, may be straightforwardly introduced in this framework and thus, compared and tested under common conditions. The requirement for the use of different macroscopic models (existing or new developments) under the same environment, imposes the avoidance of developing new complex numerical solvers. For example, the adoption of a numerical approach based on Godunov-type schemes with (exact or approximate) Riemann solvers would require the development of new different solver for each new model introduced to the framework, as for example in Coclite et al. (2005); Garavello and Goatin (2011); Wiens et al. (2013). The second objective is to explore and exploit the advantages of high-order numerical schemes for space and time discretization, in terms of computational efficiency and accuracy, to evaluate such schemes and to compare them to lower-order ones for different second-order traffic flow models.

The approach that is followed here includes the adoption of the relaxation approach proposed in Jin and Xin (1995) and is based on Finite Volume numerical discretizations of a linearized system of differential equations, which renders the methodology independent of the use of Riemann solvers (approximate or not). Within this approach, the differences between the various traffic flow models are taken into account through the corresponding flux and source term functions, plus their different parameters and upper bounds of the corresponding eigenvalues of the flux's Jacobian. Different spatial discretization schemes are incorporated in a common computational framework; a first-order upwind scheme, a second-order Monotone Upstream-centered Scheme for Conservation Laws (MUSCL) and a fifth-order weighted essentially non-oscillatory (WENO) scheme are introduced and thoroughly tested. Higher-order implicit–explicit (IMEX) Runge–Kutta (RK) schemes are also introduced for time discretization. The computational framework is formulated as to have a generalized description of variables and fluxes and to deal with different traffic flow models and future modifications.

Although low-order finite volume schemes have been successfully applied to macroscopic traffic flow modeling (we refer, for example, to Helbing and Treiber (1999); Ngoduy et al. (2004); Borges et al. (2008); Ngoduy (2013)), high-order schemes are able of reducing the numerical dissipation and producing sharper resolution in the numerical solutions. As a result, fewer discretization points are needed for a desired accuracy and the computation becomes much more efficient, compared to first-order discretization schemes. Moreover, as it will be demonstrated in following sections, specific features of the solution for some problems can be obtained only if high-order schemes are implemented. Important traffic flow conditions, such as shock waves corresponding, for instance, to queue tails, stop-and-go phenomena and cluster formations, should be accurately reproduced by a numerical scheme.

WENO schemes have been applied, mainly, to the Lighthill–Whitham–Richards (LWR), Lighthill and Witham (1955); Richards (1956), first-order macroscopic flow model and its variants, and by very few research groups so far. The fifth-order Finite Difference WENO scheme of Jiang and Shu (1996), with Lax–Friedrichs flux splitting for each equation in the system, was applied in Zhang et al. (2003) to solve the multi-class extension, proposed in Wong and Wong (2002), of the LWR model (MCLWR) with heterogeneous drivers. An extension to this work was presented in Zhang et al. (2006) to approximate an MCLWR model, modified to deal with inhomogeneous road conditions; a fifth-order accurate component-wise WENO scheme was used, which applies the Lax–Friedrichs numerical flux in the Finite Volume method and the flux splitting in the Finite Difference method. An extension of the work from Zhang et al. (2003) was presented in Zhang et al. (2009), where better estimation of the minimal characteristic speed of the MCLWR model was provided, thus improving the numerical results. In Burger et al. (2008) a family of numerical schemes was proposed for multi-species kinematic flows with discontinuous fluxes, including an MCLWR traffic flow model. These first-order schemes can be upgraded to higher-order accuracy by employing MUSCL-type techniques.

The relaxation approach introduced in Jin and Xin (1995), has found wide application in fluid dynamics problems, we refer for example to Banda (2005); Banda and Seaid (2005); Chen and Shi (2006); Delis and Katsaounis (2003); Delis and Katsaounis (2005); Delis and Katsaounis (2004); Delis and Papoglou (2008); Evje and Fjelde (2002); LeVeque and Pelanti (2001); Li et al. (2002); Seaid (2004, 2006a,b); Banda (2009), among others. The main advantage of such schemes is that neither Riemann solvers, nor the computation of eigenvalues are needed, which renders this methodology ideal for problems where an analytic expression for the eigenvalues of the systems' Jacobian matrix may not be possible or is computationally tedious to obtain, or the Riemann problems are difficult to approximate (Seaid, 2006c). However, the work on relaxation schemes for traffic flow problems includes, thus far, very few works on second- or higher-order schemes on the LWR (and variants) and low-order schemes on standard second-order traffic flow models such as the Aw and Rascle (AR)(Aw and Rascle, 2000) and the Aw–Rascle–Zhang (ARZ) models (Zhang, 2002).

In Seaid (2006c) a multi-lane LWR model was used as a test model for evaluating a relaxation scheme, developed in Eulerian and Lagrangian (characteristic) frameworks, using IMEX Runge–Kutta (RK) schemes for time integration. In the Eulerian framework a second-order MUSCL scheme was developed, and compared to the characteristic scheme and to a simple upwind one, providing in general the sharpest results. In Herty et al. (2006) relaxation schemes, utilizing a second-order central WENO scheme for spatial discretization and a third-order IMEX RK method, were used to approximate simple discrete-velocity models of traffic flows, which reduce to the LWR model in the small relaxation limit. A relaxation scheme for the MCLWR model with heterogeneous drivers was presented in Chen et al. (2009). As an extension to this work, a