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Minimum-loss network reconfiguration: A minimum spanning tree problem

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ABSTRACT

Topological reconfiguration of power distribution systems can result in operational savings by reducing the power losses in the network. In this paper, an efficient heuristic is proposed to find an initial solution for the minimum-loss reconfiguration problem with small optimality gap. Providing an initial solution for a mixed-integer programming (MIP) problem, known as "warm-start", allows for a significant speed up in the solution process. The network reconfiguration for loss reduction is mapped here into a problem of finding a minimum spanning tree (MST) in a graph, for which there are a number of efficient algorithms developed in the literature. The proposed method leads to very fast solutions (less than 1.4 s for systems up to 10 476 nodes). For the test systems considered, the solution provided by the proposed method lies within a relative optimality gap of about 2.2% with respect to the optimal solution. The existing MST algorithms guarantee the scalability of the proposed routine for large-scale distribution systems. Sensitivity factors are also employed to refine the solution to a smaller optimality gap.

1. Introduction

The tendency towards optimizing the utilization of the current infrastructure in power systems has been significantly increased in the past decade. The costs and technical difficulties associated with building new lines is a major motivation [1]. Also, reducing power losses has become a desirable objective for many distribution companies (DISCOs). Distribution systems (DS) are the last stage of the electricity network supplying energy to customers. Conventionally, DS's have been operated in radial configuration due to the simplicity of operating and protecting a radial system. Although not often, weakly-meshed configurations are sometimes encountered. Despite their extended use, radial structures are relatively vulnerable in that they have a single point of supply. Nonetheless, losing a single feeder within the whole of a DS is not potentially a threat for the entire power system operation and, therefore, the radial structure is still usually preferred.

One of the important operational problems in DS is the topological reconfiguration for loss reduction, load balancing, voltage profile improvement, etc. There are usually controllable switches in DS's using which the operator can alter system's topology by performing opening/closing actions. Since the number of switches are relatively high in practical systems, it is almost impossible for the operator to find the best topology without an optimization study. Many algorithms for DS reconfiguration have been developed, e.g., [2–4]. In these studies, the condition of radiality of the final network is normally imposed. This constraint, however, is not easy to represent as a mathematical formula, which is also recognized in [5,6].

The term "radial" refers to a configuration that includes all the nodes but has no loops. A literature review on this subject is provided in [5,6] which articulates the different approaches for imposing the radiality constraint in a configuration optimization problem. In heuristic methods, the radiality constraint is usually dealt with implicitly. Examples of heuristic methods used in the literature for network reconfiguration are Genetic Algorithms [7,8], Harmony Search [9], Simulated Annealing [10], Artificial Neural Networks [11], Plant Growth Simulation [12], Tabu Search [13], Particle Swarm Optimization [14], and Ant Colony [15]. In direct mathematical models, on the other hand, there needs to be a mathematical formulation for representing the radiality constraint. A few studies provide mathematical models for the radiality constraint, such as [16–21].

The standard constraints in an optimal DS reconfiguration problem are power flow equations, nodal voltage limits, feeders ampacity limits, and radiality. Formulating this problem using the

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traditional power flow equations leads to a mixed-integer nonlinear programming (MINLP) problem [22]. Solving large-scale MINLP problems is not practically possible due to the tremendous amount of computations required. Therefore, researchers have tried to tackle the problem using heuristic methods. The advantage of heuristic methods is their low computational complexity while being unreliable and/or suboptimal. A sensible combination of algorithms would be to use heuristics to find a good initial solution and then use this solution to initiate a deterministic mathematical optimization, e.g., [20].

There are a few deterministic mathematical formulation of the network reconfiguration problem that are in the form of a mixed-integer programming (MIP) problem. A mixed-integer conic programming formulation is proposed in [18] for network reconfiguration. It takes over 0.5 h for the algorithm in [18] to find an optimal/near-optimal solution for a network with 135 nodes. Network reconfiguration problem is formulated as a second-order cone programming in [19]. This method takes hours to provide optimal solution for a system with 880 nodes. A mixed-integer quadratically-constrained programming (MIQCP) formulation of the reconfiguration problem is proposed in [20] which is based on a linear power flow algorithm developed in [23]. The optimization problem in [20] takes about three minutes to solve the problem for a 135-node system. A mixed-integer linear programming formulation for the reconfiguration problem is developed in [24] which linearizes the current injections at each node as well as the quadratic terms in the objective function and constraints. Although all the aforementioned deterministic approaches provide many advantages in terms of flexibility of the formulation (e.g., for adding extra constraints, aiming at different objectives, knowledge about the optimality gap at every iteration, etc.), they do not take advantage of a good initial solution to start the MIP solver. Many commercial MIP solvers allow for a "warm- start", i.e. starting from a known solution, to speed up the search for the global optimum, e.g., CPLEX [25] and GUROBI [26].

In this paper, a fast method is sought which provides a suboptimal solution for the minimum-loss reconfiguration problem with a small optimality gap. This can replace the first stage of the MIP solution process that most of the commercial solvers have as a built-in routine. This routine searches for a feasible solution using some general-purpose heuristic methods. Those heuristics, however, are relatively slow and often find a solution with a large optimality gap. Providing a good initial solution to start with, the solution time can be significantly reduced [25].

The problem of finding a radial topology for a DS with minimum losses can be interpreted as finding a spanning tree in the network that also generates minimal losses. The minimum spanning tree (MST) algorithm has been adopted in [27] to find a radial topology that minimizes the energy-not-supplied. In order to assure radiality, different MST algorithms are utilized in [28] (Kruskal algorithm) and [29] (Prim algorithm) within a Genetic Algorithm-based search. However, the only use of the MST algorithms in the referred studies is to help finding a radial configuration, not to find the minimum losses.

In this paper, the problem of minimum-loss DS reconfiguration is mapped into a MST problem based on some assumptions. The main assumption here, based on engineering knowledge, is that the meshed network is a good solution (if not the best) for loss minimization when the radiality constraint is relaxed. The same assumption has been made in [21] with a slight difference. In [21], it is assumed that the meshed network generates the least possible losses, which may not be always true as shown in this paper. However, the difference between the losses in the meshed network and the best-possible network configuration is negligible, as is also shown later in this paper. The second assumption in the present paper is that reconfiguring the network in order to reduce the losses will implicitly improve the voltage profile. This assumption has been previously validated in [4], and is also validated in this paper on various test systems. The voltage profile can also be modified by adjusting capacitor banks, voltage regulators, and transformers tap positions, as is done in [20], outside of the reconfiguration process.

Based on the assumptions mentioned in the previous paragraph, the problem of DS reconfiguration for loss reduction is defined as "finding a spanning tree that imitates, as closely as possible, the same flow pattern as the meshed network". The idea of starting from a meshed network and opening switches sequentially has been proposed in [4], called DISTOP. There are two main differences between DISTOP and the proposed method in this paper. The first difference is that after every switching action, a new power flow solution is required in DISTOP, while only one power flow solution is sufficient for the proposed algorithm in this paper. The other difference is how the algorithms check whether a configuration is radial. It is stated in [4] that the switches are opened one after another until the network becomes radial. However, it is not explicitly explained how to check the radiality. In many cases, disconnecting a switch that carries the minimum current leads to a disconnected network. It is not trivial to check if the network remains connected after opening a particular switch. The proposed algorithm in this paper, on the other hand, is guaranteed to provide a radial configuration.

The proposed heuristic method here has the advantage of using a fast and robust algorithm that also finds a high-quality solution (with small optimality gap). The efficient MST algorithms developed in the literature such as Kruskal [30] and Prim [31] algorithms can be employed to solve the formulated problem. A refinement to the solution is done by sensitivity analysis around the neighborhood of the candidate tie switches. In order to do that, line outage distribution factors (LODF) are derived based on the linear power flow formulation in [23].

The rest of this paper is organized as follows. In Section 2, a brief background on graph theory and the MST problem is presented. In Section 3, the DS reconfiguration problem is converted into a MST problem. Section 4 presents the application of the proposed method to various test cases. The main findings of this study are summarized in Section 5.

2. Background: minimum spanning tree

A spanning tree is a subgraph of an undirected graph that contains all the vertices (i.e. it is connected) and has no cycles (has no loops). The MST in a weighted undirected graph is the subgraph that is a spanning tree, and the sum of its weights is the minimum possible. This problem is well-addressed in the literature under the name "minimum spanning tree" (MST) and there are efficient algorithms such as Kruskal [30] and Prim [31] to solve the problem. Besides, there are parallel algorithms to solve the MST problem even faster, e.g., [32]. By negating the weights, the algorithms for MST will find the maximum spanning tree [33]. The time complexity (the amount of time taken by the algorithm to run as a function of the network size) of a version of Prim's algorithm which is suitable for sparse graphs is $\mathcal{O}(e \log v)$, which is similar to the time complexity of Kruskal's algorithm.

Prim's algorithm is used in this paper to find the MST. There is basically no advantage of using Prim's algorithm over Kruskal's, and one may choose the other. It is important to emphasize that these algorithms provide the optimal solution to the MST problem. The structure of the Prim's algorithm for a weighted undirected graph $G(V, E)$ with V vertices and E edges is summarized in the following steps.