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Robust constrained control of uncertain macroscopic fundamental diagram networks

Jack Haddad [*]

Technion – Israel Institute of Technology, Technion Sustainable Mobility and Robust Transportation (T-SMART) Laboratory, Technion City 32000, Haifa, Israel

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ABSTRACT

This paper considers modeling and control of uncertain Macroscopic Fundamental Diagram (MFD) systems for multiple-region networks. First, the nonlinear vehicle conservation equations based on MFD dynamics, presented in earlier publications, are transformed to linear equations with parameter uncertainties. The parameter uncertainties include the destination decomposition fractions, that are difficult to estimate in reality. Then, the uncertain linear model is utilized to design a robust feedback controller by an interpolation-based approach. This approach (i) guarantees robustness against all parameter uncertainties, (ii) handle control and state constraints, and (iii) present a computationally cheap solution. The main idea is to interpolate between (i) a stabilizing outer controller that respects the control and state constraints and (ii) an inner robustly stable controller designed by any method. The robust control is further challenged to deal with different relative locations of reference accumulation points on the MFD diagrams. Numerical results for a two-region system show that the uncertain linear model can replace the nonlinear model for modeling and control. Moreover, the robust control law is presented as implicit and explicit solutions, where in the implicit case one linear programming (LP) problem is solved at each time instant, while in the explicit case, the control law is shown as a piecewise affine function of state. Finally, a comparison between the interpolating controller and other controllers in the literature is carried out. The results demonstrate the performance advantages from applying the robust interpolating controller.

1. Introduction

Macroscopic Fundamental Diagrams (MFDs) have been widely investigated by many researchers. Empirical and simulation studies have provided observation and theoretical elements for the existence of well-defined MFDs for homogeneous urban networks with small variance of link densities Godfrey (1969), Daganzo (2007), Geroliminis and Daganzo (2008), Geroliminis and Sun (2011b), Buisson and Ladier (2009), Ji et al. (2010), Mazloumian et al. (2010), Daganzo et al. (2011), Mahmassani et al. (1987), and Olszewski et al. (1995). It is shown that the MFD can provide a unimodal, low-scatter relationship between network vehicle density (veh/km) and network space-mean flow or outflow (veh/hr) for different network regions, if congestion is roughly homogeneous in the network.

These research findings encourage researchers to develop MFD-based models, that aim at (i) modeling the aggregated traffic flow characteristics and (ii) designing control strategies at large-scale urban networks.

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[*] Tel.: +972 778871742.
E-mail address: jh@technion.ac.il

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With respect to the MFD modeling, vehicle conservation equations based on MFD dynamics have been developed for *one urban region* networks in Daganzo (2007), Geroliminis and Daganzo (2008), Haddad and Shraiber (2014), Keyvan-Ekbatani et al. (2012), and for heterogeneous networks partitioned into *multi homogeneous urban regions* in Haddad and Geroliminis (2012), Geroliminis et al. (2013), Aboudolas and Geroliminis (2013), Hajiahmadi et al. (2015). The main difference between these models is that in Haddad and Geroliminis (2012), Geroliminis et al. (2013), Hajiahmadi et al. (2015) the regional accumulations in the dynamic vehicle conservation equations are decomposed based on destinations. In Haddad et al. (2013), the urban MFD-based model is extended to a network structure which consists of freeways and urban roads, where a simple route choice between urban and freeway routes is integrated. In Xiong et al. (2015) modeling agents' en-route diversion behavior under information provision has been introduced, where the dynamic behavioral responses and network performance are represented by MFDs. Moreover, in Geroliminis et al. (2014), a three dimensional MFD model is developed for mixed bi-modal urban traffic, which consists cars and buses sharing the same network infrastructure, and in Haddad (2015), boundary aggregated queue dynamics for two regions are integrated in the MFD-based model. Modeling the dynamics of heterogeneity with a parsimonious model has been developed in Ramezani et al. (2015) and Mahmassani et al. (2013). Recently, in Zhang et al. (2015) integrating a Cell Transmission Model with the MFD for urban networks is proposed and its effects analyzed.

With respect to control, different control strategies utilizing the concept of the MFD have been introduced for urban networks. Perimeter control strategies, i.e. manipulating the transfer flows at the perimeter border of the urban region, have been introduced for *single-region* cities in Daganzo (2007), Keyvan-Ekbatani et al. (2012), and Keyvan-Ekbatani et al. (2013), and for *multi-region* cities in Haddad and Geroliminis (2012), Geroliminis et al. (2013), and Aboudolas and Geroliminis (2013). In Haddad et al. (2013), different levels of coordination between freeways and urban roads have been proposed. Recently, Hajiahmadi et al. (2015) have introduced a combination of perimeter control for the boundary with switching timing plans for each region. The different timing plans of individual intersections result in different shapes of MFDs. Moreover, route guidance strategies with the utilization of MFD have been studied in Knoop et al. (2012) for grid networks without traffic lights, and Gayah and Daganzo (2011) also studied simple routing strategies for two-bin networks. Daganzo and Geroliminis (2008), Geroliminis and Boyacı (2012), and Zhang et al. (2013) have shown that traffic-responsive signal control strategies and different signal settings can change the shape of the MFD and the critical accumulations.

With respect to control approaches, Model Predictive Control (MPC) approach has been used to solve the optimal control problems in Geroliminis et al. (2013), Haddad et al. (2013), and Hajiahmadi et al. (2015), while a classical feedback control approach has been implemented in Keyvan-Ekbatani et al. (2012), and Aboudolas and Geroliminis (2013). The developed MPC solution for the different models has performed well for different levels of demand and errors in the MFD's shape, given that the prediction dynamic models are sufficiently accurate. On the other hand, the designed controllers by the classical approach operate according to the feeded information without the need to predict the near-future dynamics or demand, which might be preferable for real practical implementation. Moreover, both works Keyvan-Ekbatani et al. (2012) and Aboudolas and Geroliminis (2013) do not allow direct consideration of the control constraints, but impose them after the design process, e.g. adjusting or fine-tuning the controller gains.

Introducing robust perimeter control strategies for uncertain MFD networks is challenging. Different types of uncertainty can be integrated in the MFD-based model parameters. E.g. heterogeneous networks might not have a well-defined MFD, especially in the decreasing part, Daganzo et al. (2011), Buisson and Ladier (2009), and Geroliminis and Sun (2011a). Partitioning such a network into homogenous regions can result in well-defined shape MFD with low scatter, as shown in Ji and Geroliminis (2012) and Ji et al. (2014). Therefore, in order to deal with scattered MFD in case of heterogeneity, one can integrate parameter uncertainties in the model to cope with such difficulties. Other types of uncertainty can be also integrated in the model parameters, e.g. Ampountolas et al. (2014) have introduced uncertainty for different traffic compositions, i.e. percentages of buses, in the three dimensional MFD.

Recently in Haddad and Shraiber (2014), a robust perimeter controller is designed for *an urban region* with the MFD representation including MFD uncertainty, which includes scatter of flows for the same accumulation. The designed PI-controller stabilizes the linearized system against all uncertainties. The robust control in Haddad and Shraiber (2014) is designed based on the principles of Quantitative Feedback Theory, Houpis et al. (2006). Moreover, the control constraint is integrated in the closed-loop control with the help of the so-called describing function. Note that the describing function should be carefully chosen to guarantee satisfying the control constraint. The robust feedback control in Ampountolas et al. (2014) for the three dimensional MFD model is designed via Linear Matrix Inequalities (LMI) optimization, but without integrating constraints.

While the desired state in the one urban region system is known in advance (given the MFD shape), for the system with multiple urban regions the desired accumulation points are not well known. Therefore, a challenging control question is how to regulate a network with multiple regions. Note that the accumulation references can be a priori given set points, or other desired constant accumulation points, or accumulation trajectories which might vary with time.

Motivated by the results in Haddad and Shraiber (2014), in this paper, we aim at designing robust constrained control of uncertain MFD networks. With respect to modeling, we aim at reformulating the nonlinear MFD model for *multi-region* MFD systems presented in Geroliminis et al. (2013), Hajiahmadi et al. (2015), and Haddad and Geroliminis (2012) to a linear model with parameter uncertainties, where the uncertainties include the destination decomposition fractions of the regional accumulations in the dynamic equations. These fractions are difficult to estimate them in real-time process, but can be