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# Traffic signal control and route choice: A new assignment and control model which designs signal timings

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ABSTRACT

This paper outlines a dynamical assignment and control model which calculates stage green-times for a signal-controlled network. In the dynamical model route flows, bottleneck delays and stage green-times all change simultaneously over iterations. It is shown that if the initial problem is feasible and the $P_0$ policy is utilised to move stage green-times then convergence is certain in the vertical queueing case. The model makes some reasonable systematic allowance for travellers' route choices by encouraging congestion-reducing route choices in the future; the model (in generating signal timings) maximises network capacity (taking account of route choices) when queueing is vertical. Blocking back however appears difficult to deal with (while retaining the convergence guarantee) and represents an area for further study. The model may be used to design fixed-time or time of day signal timings; it may also be used to pre-prepare timings for rapid implementation in case of a predictable incident; finally if computation speeds are high enough the method may possibly be used responsively, in real-time.



## 1. Introduction

### *1.1. The main purpose and contributions of the paper*

The main purpose of this paper is to state a new dynamical traffic assignment and control model which

(i) designs fixed time or time of day signal timings for average days and
(ii) designs suitable signal timings for at least some predictable incidents.

In both cases the designed timings take some reasonable systematic account of future route choices. An associated purpose is to state some options for responsive control policies which emerge from a consideration of the dynamical "design" models presented in this paper.

The obvious scarce commodity in congested networks is "junction capacity"; an economical routing pattern should consume as little as possible of this and so signal timings generated for time-of-day application, for application when there is an incident and for responsive application should all ideally encourage future congestion-reducing route switches (toward routes which utilise less junction capacity). *At the moment, in practice, traffic signal timings are designed or optimised without systematically seeking to influence route choices beneficially.*

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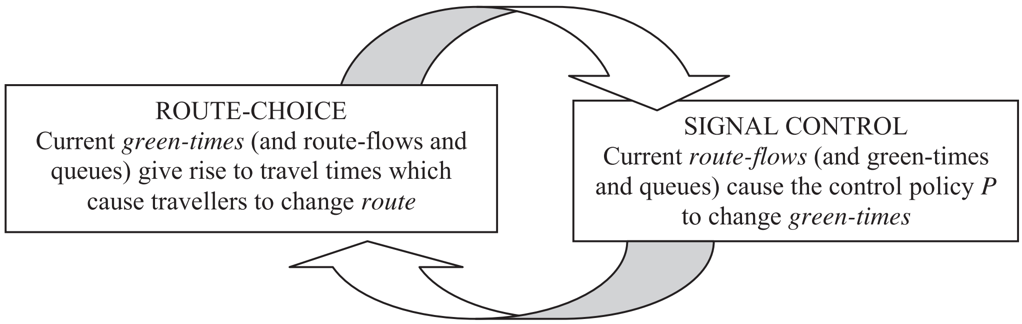


**Fig. 1.** A dynamical system arising when a responsive control system following a control policy $P$ is utilised either in reality or within a model. The loop may be thought of as being traversed indefinitely.

The central assignment and control model developed in this paper follows the natural dynamical system illustrated in Fig. 1. It is shown how such a model may be utilised to design signal timings, by suitably choosing control policy $P$ and by running a suitable dynamical model following Fig. 1 to approximate equilibrium.

A simple model is used at the start of the paper to illustrate the main ideas. The main contributions in the paper are as follows.

1. A general queueing version of policy $P_0$ is stated in Section 2.8; the policy moves green-time from less to more pressurised stages with a particular definition of stage pressure.
2. A proof is given (in Section 2.10) that the $P_0$ control policy does maximise the capacity of certain general networks (with vertical queueing or limited spatial queueing): by this we mean that (for any feasible demand) if policy $P_0$ is utilised then there is an equilibrium with queueing consistent with the policy. (See Definition 1). Such a proof is in general impossible with other policies.
3. A specific convergent dynamical system following Fig. 1 is stated (in Sections 3–6); this includes dynamical models of route flows, vertical bottleneck delays and green-times (following a dynamical version of the $P_0$ policy). The dynamical system converges to the set of equilibria consistent with $P_0$.

The networks considered initially have a steady demand and vertical queueing; these networks are essentially dynamic (as queueing delays are represented) but with a constant instead of varying demand. Such networks are often called "quasi-dynamic networks"; see Bliemer et al. (2012) and Nesterov and de Palma (2003). (Some limited extensions to allow for spatial queueing are considered in this paper too.)

The capacity-maximisation proof in Section 2.10 relies essentially on showing that the control policy $P_0$ arises naturally within the Kuhn-Tucker conditions for the following optimisation problem: *find green times and route flows which meet the given demand, are within the capacity restrictions of the network and minimise total uncongested travel cost.*

### *1.2. An outline of the paper*

This paper considers a variety of assignment and control models, leading up to that in Fig. 1. The basic ideas here were first put forward in Smith et al. (1987). Flows, signal timings and usually bottleneck delays are represented; so any signal timings arising as outputs from any of the models described here must be at least partially routeing-aware. The paper suggests how such models may be utilised to design time of day signal timings and to help suggest reasonable responsive signal control strategies:

(i) An "equilibrium" of the assignment and control models yield fixed time signal settings.
(ii) Certain of the models have dynamics which suggest responsive control strategies.

Section 2 considers certain assignment and control models *in a very simple initial example network*. All the initial signal timings generated have the property that they are consistent with equilibrium route choices; they allow the full capacity of the simple network to be realised at a user equilibrium – *they are capacity maximising*. These initial strategies may all be written in a general form, so that they may be applied (in principle) to a general network; these initial strategies maximise the travel capacity of a general quasi-dynamic network with vertical queueing. (This is a network with queues where the vehicles may be thought of as being very short and where demand is constant.)

It is shown in Sections 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 how the particular control policy $P_0$ arises; essentially as part of the Kuhn-Tucker conditions of a distance travelled minimisation problem.

Section 2.9 considers briefly connections with three other signal setting methods.