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Control of a lane-drop bottleneck through variable speed limits

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ABSTRACT

The discharging flow-rate of a lane-drop bottleneck can drop when its upstream is congested, and such capacity drop leads to additional traffic congestion as well as safety threats. Even though many studies have demonstrated that variable speed limits (VSL) can effectively delay and even avoid the occurrence of capacity drop, there lacks a simple approach for analyzing the performance of a VSL control system. In this study, we formulate the VSL control problem for the traffic system in a zone upstream to a lane-drop bottleneck based on two traffic flow models: the Lighthill–Whitham–Richards (LWR) model, which is an infinite-dimensional partial differential equation, and the link queue model, which is an ordinary differential equation and approximates the LWR model. In both models, the discharging flow-rate is determined by a recently developed model of capacity drop, and the upstream in-flux is regulated by the speed limit in the VSL zone. We first analytically study the properties of the control system with the link queue model. For an open-loop control system with a constant speed limit, we prove that a constant speed limit can introduce an uncongested equilibrium state, in addition to a congested one with capacity drop, but the congested equilibrium state is always exponentially stable. Then we apply a feedback proportional-integral (PI) controller to form a closed-loop control system, in which the congested equilibrium state and, therefore, capacity drop can be removed by both I- and PI-controllers. Both analytical and numerical results show that, with appropriately chosen controller parameters, the closed-loop control system is stable, effect, and robust. Finally, we show that the VSL strategies based on I- and PI-controllers are also stable, effective, and robust for the LWR model. Since the properties of the control system are transferable between the two models, we establish a dual approach for studying the control problems of nonlinear traffic flow systems. We also confirm that the VSL strategy is effective only if capacity drop occurs. The obtained method and insights can be useful for future studies on other traffic control methods and implementations of VSL strategies in lane-drop bottlenecks, work zones, and incident areas.

1. Introduction

An important characteristic of various freeway bottlenecks is that, when a bottleneck becomes active; i.e., when one of the upstream branches is congested but the downstream branch is not, the maximum discharging flow-rate of the bottleneck can drop to a lower level than the downstream capacity. In other words, there can be two distinctive capacities for

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uncongested and congested traffic at such bottlenecks. This is the so-called two-capacity or capacity-drop phenomenon of active bottlenecks, in which “maximum flow rates decreases when queues form” (Banks, 1990, 1991b; Hall and Agyemang-Duah, 1991). Such capacity drop has been observed at merges, tunnels, lane drops, curves, and upgrades, where the bottlenecks cannot provide sufficient space for upstream vehicles (Chung et al., 2007; Kim and Cassidy, 2012). In free flow, the bottleneck’s capacity can be fully utilized, and the maximum flow-rate for the bottlenecks can reach around 2300 vphpl, which is about the same as the capacity of a homogeneous road lane without bottlenecks (Federal highway administration, 1985; Hall and Agyemang-Duah, 1991). However, with excessive traffic demands the traffic system can break down, the maximum discharge flow-rate drops below the free-flow capacity. It was shown that such dropped capacity is stable, although interactions among several bottlenecks can cause fluctuations in discharging flow-rates (Kim and Cassidy, 2012). Depending on the geometry of a bottleneck, the magnitude of capacity drop is in the order of 10% (Persaud et al., 1998; Cassidy and Bertini, 1999; Bertini and Leal, 2005; Chung et al., 2007). In (Persaud et al., 1998, 2001), traffic breakdown and capacity drop were found to be related to the upstream traffic demand randomly. Also closely associated with capacity drop is the existence of discontinuous flow-density relations in fundamental diagrams. For examples, in (Edie, 1961), discontinuity was observed in the flow-density relation for the Lincoln tunnel; in (Drake et al., 1967; Koshi et al., 1983), a flow-density relation of the reverse-lambda shape was calibrated; more studies have confirmed the existence of discontinuous fundamental diagrams (Payne, 1984; Hall et al., 1992). In (Hall and Agyemang-Duah, 1991), it was shown that such discontinuous fundamental diagrams generally arise in the congested area of an active bottleneck and suggested that the discontinuity is caused by the capacity drop phenomenon.

In the literature, capacity drop has been modeled by discontinuous fundamental diagrams (Lu et al., 2008, 2009). However, empirical observations suggest that fundamental diagrams are still continuous in steady-state traffic flow (Cassidy, 1998), and theoretical analyzes show that unrealistic, infinite characteristic wave speeds can arise from a discontinuous flow-density relation (Li and Zhang, 2013). In contrast, in (Jin et al., 2013) a new kinematic wave model of capacity drop was proposed. In the new model, fundamental diagrams are still continuous, but the entropy condition for uniquely solving the system of hyperbolic conservation laws is discontinuous. The new entropy condition is a junction flux function used to prescribe discharging flow-rates from the upstream demand and downstream supply. Therefore, the model can be easily discretized and incorporated into the Cell Transmission Model (CTM) (Daganzo, 1995; Lebacque, 1996). Furthermore, the model was shown to be well defined analytically, and the new junction flux function was shown to be invariant (Lebacque and Khoshyaran, 2005; Jin, 2012b). Therefore, the junction flux can also be incorporated into the link queue model (Jin, 2012c), which is a system of ordinary differential equations but approximates the kinematic wave model. In addition, with empirical observations it was shown that the new model can explain the existence of discontinuous flow-density relation in steady traffic states.

Clearly, a drop at the active bottleneck’s capacity can reduce the total discharging flow-rate of the whole corridor and prolong vehicles’ travel times (Daganzo, 1999). That the capacity of a road network may drop substantially when it is most needed during the peak period has been a baffling nature of freeway traffic dynamics (Papageorgiou and Kotsialos, 2002). Therefore, an important motivation and theoretical foundation for developing ramp metering, variable speed limits, and other control strategies is to avoid or delay the occurrence of capacity drop (Banks, 1991a; Papageorgiou et al., 1991, 1997; Cassidy and Rudjanakanoknad, 2005; Papageorgiou et al., 2005, 2007).

Variable speed limits (VSL) have been implemented to improve the throughput of a traffic system since 1960s (Greenberg and Daou, 1960). Recently, feedback-based VSL strategies have been developed and simulated with a macroscopic second-order traffic flow model for lane-drop bottlenecks in (Carlson et al., 2011, 2013). In addition, there have been many studies on integrated control strategies based on both VSL and ramp metering. In (Hegyi et al., 2005), network-wide coordination of VSL and ramp metering was studied within the framework of model predictive control based on a higher-order simulation model. In (Zhang et al., 2006), the ALINEA ramp metering algorithm is coupled with a VSL strategy to control a stretch of freeway. In (Allaby et al., 2006), microscopic simulations based on PARAMICS are used to study the impacts of VSL on both safety and efficiency of the eastbound Queen Elizabeth Way (QEW) located near Toronto, Canada, but it showed that travel times can be increased by VSL, due to the temporal and spatial limitations of VSL algorithms. In (Papageorgiou et al., 2008), it was shown that mainline freeways VSL can impact fundamental diagrams and could increase discharging flow-rates in congested traffic. In (Lu et al., 2010), an integrated variable speed limit and ramp metering strategy was proposed, and it was shown that by creating a discharging section right before a bottleneck can increase discharging flow-rate. In (Carlson et al., 2010), with a macroscopic second-order traffic flow model, it was shown that VSL have similar effects as ramp metering; coordination of VSL control and ramp metering is solved as a discrete-time optimal control problem for large-scale networks. In (Cascetta et al., 2011), it was shown that speed enforcement can reduce bottleneck congestion and travel time with empirical evidence.

In this study, we are interested in the variable speed limit (VSL) strategy applied to tackle the capacity drop at a lane-drop bottleneck as shown in Fig. 1, which can be caused by lane closures, work zones, or incidents. The capacity in the downstream section, C, is determined by systematic lane changes inside zones 1 and 2 (Jin, 2010a,b, 2012a; Gan and Jin, 2013). When the traffic demand, $d(t)$, in zone 0, which was referred to as the “acceleration zone” in (Carlson et al., 2010), is higher than C, a queue forms in zone 1, the capacity drops by Δ to $C(1-\Delta)$, and vehicles accelerate to the free-flow speed in zone 2. However, when the VSL zone is introduced upstream to zone 0, its demand, $d^{-}(t)$, can be controlled by the variable speed limit, $u(t)$. Then $d(t)$ in zone 0 can be restricted, so that zones 1 and 2 will be clear and the total discharging flow-rate can reach C.