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Consistency in multi-vehicle inventory-routing

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ABSTRACT

Inventory-routing problems (IRPs) arise in vendor-managed inventory systems. They require jointly solving a vehicle routing problem and an inventory management problem. Whereas the solutions they yield tend to benefit the vendor and customers, solving IRPs solely based on cost considerations may lead to inconveniences to both parties. These are related to the fleet size and vehicle load, to the frequency of the deliveries, and to the quantities delivered. In order to alleviate these problems, we introduce the concept of consistency in IRP solutions, thus increasing quality of service. We formulate the multi-vehicle IRP, with and without consistency requirements, as mixed integer linear programs, and we propose a matheuristic for their solution. This heuristic applies an adaptive large neighborhood search scheme in which some subproblems are solved exactly. The proposed algorithm generates solutions offering a good compromise between cost and quality. We analyze the effect of different inventory policies, routing decisions and delivery sizes.

1. Introduction

In vendor-managed inventory (VMI) systems, the replenishment and distribution making process is centralized at the supplier's level. The application of this policy leads to an overall reduction of logistics costs (Lee and Seungjin, 2008) and is often described as a win–win situation. By deciding when and how much to deliver to their customers, suppliers can reduce their overall distribution costs by smoothing their delivery schedules and by efficiently combining in the same period visits to customers that are geographically close to one another. Customers also benefit by saving on ordering costs.

Optimizing a VMI system requires the solution of a difficult combinatorial optimization problem called the inventory-routing problem (IRP). The IRP combines inventory management and routing decisions over several periods into the same problem. Typically, the supplier is free to decide the size of the delivery to each customer, being constrained only by the inventory holding capacity at each site and by the capacities of its vehicles. This general delivery policy is called maximum level (ML). Several heuristics (Archetti et al., 2012; Bertazzi et al., 2002; Coelho et al., 2012) and an exact algorithm (Archetti et al., 2007) have been proposed for the single vehicle case of this problem. A large number of variants of the IRP have arisen since this problem was first introduced by Bell et al. (1983). Literature reviews can be found in Andersson et al. (2010) and Cordeau et al. (2007).

Whereas VMI policies are clearly beneficial from a system's perspective, they may sometimes result in inconveniences both to the supplier and to the customers. This is the case, for example, when very small deliveries take place on consecutive days, followed by a very large delivery, after which the customer is not visited for a long period. Another example, this time undesirable for the supplier, is that it could be optimal to dispatch a mix of almost full and almost empty vehicles, which does not yield a proper load balancing and may irritate some drivers.

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Companies need not only provide cost effective solutions to their customers, but also high quality service. This can be partly achieved by incorporating quality of service features in IRP solutions, which should yield a competitive advantage. To this end, we introduce the concept of *consistency* in the IRP in order to reflect some common quality of service standards. This can be achieved, for example, through the application of workforce management policies (Barlett and Ghoshal, 2002; Groër et al., 2009; Smilowitz et al., 2012). Thus, one would expect that regularly assigning the same driver to customers will help create a bond that can benefit both parties. Drivers will gain an increased familiarity with the region and the customer sites assigned to them, and will thus develop a more personal rapport with the customers. Another example of consistency is the spacing of deliveries to customers. To ensure smoother operations, visits should ideally be spread out evenly over the planning horizon. This type of requirement is often modeled as constraints in the context of the periodic vehicle routing problem (VRP) (Christofides and Beasley, 1984; Francis et al., 2008) but it has not yet been imposed in the IRP. Finally, the quantities delivered to customers can also be controlled in order to avoid large variations over time, which are negatively perceived by customers (Beamon, 1999). In this paper, we consider six different consistency features in IRP solutions:

1. *Quantity consistency:* any delivery performed to a customer must lie within certain customer-dependent intervals, to avoid large variations. From the customers' point of view, delivery size is important. If deliveries are too small, then customers will have to receive frequent visits, which is inconvenient and time-consuming. Deliveries that are too large may create congestion in the warehouse.
2. *Vehicle filling rate:* a vehicle can only be used if its filling rate lies within a certain interval.
3. *Order-up-to (OU) policy:* this is a common IRP constraint (see e.g. Archetti et al., 2007, 2011, 2012; Bertazzi et al., 2002; Coelho et al., 2012) which can be viewed as a consistency feature. It states that whenever a visit is performed to a customer, the delivery should fill the customer's inventory capacity.
4. *Driver consistency:* this requirement means that each customer is assigned to one driver.
5. *Driver partial consistency:* one shortcoming of the previous feature is that it may cause a vehicle to serve very few customers and thus its effect may be very costly. We relax this rule by allowing some deliveries not to be subject to it.
6. *Visit spacing:* we impose a minimum and a maximum interval between two consecutive visits to the same customer.

Some of these features (e.g. 1 and 6) should depend on the stability of the demand. If the demand is highly variable, customers would expect deliveries to be variable as well, because consistency would then make little sense. However, it is known (Barrat, 2003; Olson and Xie, 2010) that the application of VMI requires some demand stability, which legitimates the consistency features we propose. It is also relevant to note that some of the six consistency features cannot be used in combination with some others. For example, 4 is stronger than 5; the OU policy cannot always be enforced if features 1, 2, or 6 are implemented; other combinations of the consistency features, like 1 and 2, may yield infeasible solution for some parameter values. The choice, application and parameters regulating each consistency feature should be the object of discussion and negotiation between customers and the supplier, as is the case of any VMI strategy (Erhun and Keskinocak, 2011).

The concept of driver consistency has already been applied by Groër et al. (2009) to a version of the VRP in which customers receive visits on prespecified days. The authors have proposed a model ensuring that the same customer is always served by the same driver as a means of improving quality of service, but the application of this constraint to the IRP is new and more complicated because the visit days are endogenous and because of the inventory management issues involved.

We model and solve the *basic* multi-vehicle version of the problem (MIRP) considered in Archetti et al. (2007, 2012) and Bertazzi et al. (2002) to which we incorporate the consistency features just described. Although the MIRP has previously been studied, the variety of assumptions has left a gap in the literature in the sense that one cannot find benchmarks to a common version of the problem. For instance, to cite some recent contributions to the MIRP literature and their different assumptions, Abdelmaguid and Dessouky (2006) allow backorders and use a non-linear transportation cost function which depends on the quantity delivered, Dauzère-Pérès et al. (2007) have studied the stochastic version of the problem, and Yu et al. (2008) did not include supplier inventory costs. Here we define and solve benchmark instances of the MIRP derived from those of Archetti et al. (2007, 2012) for the single vehicle case, with and without consistency requirements. Our algorithm can also solve the consistent VRP with capacity constraints.

The main scientific contribution of this paper is to add consistency requirements to the basic MIRP and to develop a matheuristic for this version of the MIRP, called the *consistent* MIRP. The remainder of the paper is organized as follows. In Section 2 we formally describe the basic MIRP and we present a mixed-integer linear programming formulation for it and for the consistent MIRP. Section 3 describes our algorithm which combines adaptive large neighborhood search and the exact solution of mixed integer linear programs. This algorithm can solve the basic MIRP and the consistent MIRP defined by any meaningful combination of the six features just introduced. This is followed by the results of extensive computational experiments in Section 4, and by conclusions in Section 5.

2. Formal problem description and mathematical models

We now formally introduce the basic MIRP. The problem is defined on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \{0, \ldots, n\}$ is the vertex set and $\mathcal{A} = \{(i,j) : i,j \in \mathcal{V}, i \neq j\}$ is the arc set. Vertex 0 is a depot at which the supplier is located and the vertices of