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A discrete traffic kinetic model – integrating the lagged cell transmission and continuous traffic kinetic models

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ABSTRACT

Continuous traffic kinetic models are difficult to solve because of the occurrence of integro-differential equations in the models. In this paper, we formulate a discrete traffic kinetic model by extending the cell transmission mechanism, which can capture not only the number of vehicles, but also the velocity probability distribution. The variation in the velocity probability distribution is modeled on the basis of vehicle conservation and cell transmission to avoid integro-differential terms. An example with a discontinuous initial density is analyzed to demonstrate the validity of the proposed discrete traffic kinetic model. From the evolution curve of the velocity probability distribution, we can see that the proposed model can be used to describe the diffusion process of vehicles from a high density to a low density section.

1. Introduction

Following the seminal work of Prigogine, a kinetic theory of traffic flow has been developed that is similar in spirit to the kinetic theory of gases and results in Boltzmann-type equations. The Prigogine–Herman (P–H) traffic kinetic theory (Prigogine and Herman, 1971) includes a velocity distribution function $f(x, v, t)$, which is introduced and determined by considering three processes (i.e., interaction, relaxation, and adjustment process) in traffic flow. Munjal and Pahl (1969) reviewed the hypothesis and assumptions of the P–H model, and validated the slowing-down term in the presence of platoons. However, among the interaction, relaxation, and adjustment terms in the model, only the interaction term has been derived analytically. The relaxation term appears to be postulated based on intuitive grounds, and the adjustment term is introduced to include a follow-the-leader effect in the Boltzmann-type equation. Paveri-Fontana (1975) improved the P–H model by introducing the joint distribution of the velocity and desired velocity. From the late 1970s to the late 1980s, few studies of traffic kinetic models were published.

Since the 1990s, however, there has been renewed research interest in traffic kinetic models, which can be divided into four areas. First, the P–H traffic kinetic model has been further improved. For example, Nelson (1995) improved this model by analytically deriving speeding-up interactions using correlation and mechanical models. This improvement revised the phenomenological relaxation term of Prigogine and Herman (1971), which is intuitively modeled and not analytically derived. Klar and Wegener (1997) employed Nelson's ideas to construct a kinetic model of vehicular traffic, which takes into account vehicle length. Helbing (1996) derived a gas-kinetic traffic equation based on the basic laws of the acceleration and

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interaction of vehicles. Helbing and Treiber (1998) proposed a gas-kinetic traffic model to explain hysteretic transition. Hoogendoorn and Bovy (2001) introduced the generalized phase-space density to generalize the Paveri-Fontana model and developed a traffic kinetic model that considers both discrete attributes, including user-class, roadway lane, and destination, and continuous attributes, including velocity and desired velocity. Nelson (2003) proposed three benchmarks for kinetic models: the kinetic equation solution should be bimodal at high densities; the corresponding traffic flow model should display observed scattering at high densities; and the first-order Chapman–Enskog solution should be an arguably reasonable improvement on the Lighthill–Whitham model (1955). Ngoduy (2006) developed the gas-kinetic equations with discontinuities for interrupted traffic flow of weaving sections, and used moment method to obtain the corresponding macroscopic model.

Second, a macroscopic traffic hydrodynamic model has been derived based on the traffic kinetic model. A macroscopic gas-kinetic-based traffic model with a non-local interaction term was derived from a microscopic model of vehicle dynamics (e.g., Treiber et al., 1999; Helbing et al., 2001). Some studies show that hydrodynamic models of traffic flow can be obtained via asymptotic expansions of equilibrium solutions of kinetic equations. For example, Nelson and Sopasakis (1999) used Chapman–Enskog-type expansions to obtain zero- and first-order model equations of traffic flow, while Sopasakis (2003) used Hilbert expansions to obtain zero- and first-order models. Mendes and Velasco (2008) used the Grad's moment method to generate a distribution function based on the homogenous state solution of the Paveri-Fontana equation to construct a macroscopic model.

The third research area concerns the development of a general modeling framework describe multiple attributes and multi-dimensional particle movement (e.g., Hoogendoorn and Bovy, 2001).

The fourth area involves the formulation of discrete traffic kinetic models with the aim of solving the problem of the lack of an efficient algorithm in continuous traffic kinetic theory because of integro-differential terms. Meng et al. (2008) formulated a discrete lattice Boltzmann model based on traffic kinetic theory, which can capture the metastability and stop-and-go phenomena of traffic flow. Meng et al. (2008) extended their lattice Boltzmann model for road traffic to model urban traffic networks. One deficiency of the model is that the simplification of the interaction and relaxation terms as a uniform relaxation term toward the local equilibrium velocity distribution has no analytical derivation. Delitala and Tosin (2007) proposed a kinetic model with discrete velocities for vehicular traffic flow, using the mathematical kinetic theory approach, which was developed to model biological systems. Bonzani and Mussone (2008) identified the parameters of a discrete traffic kinetic model using experimental data. Bonzani and Gramani Cumin (2008) applied discrete kinetic theory to model vehicle distribution along different lanes of a spatially homogeneous multilane highway.

This paper contributes to the fourth research area. It proposes a discrete traffic kinetic model that incorporates a lagged cell-transmission model (LCTM) to overcome the problem of the lack of an efficient algorithm for solving a continuous traffic kinetic model because of the occurrence of integro-differential terms.

2. Classic P–H traffic kinetic model

The kinetic model of vehicular traffic developed by Prigogine and Herman (1971) describes the space–time evolution of the velocity distribution. In terms of driver behavior at low density, it can predict the velocity distribution at an arbitrary density.

Prigogine and Herman's kinetic equation is

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial t}\right)_{\text{relaxation}} + \left(\frac{\partial f}{\partial t}\right)_{\text{interaction}} + \left(\frac{\partial f}{\partial t}\right)_{\text{adjustment}}. \tag{1}$$

This equation describes the time evolution of the velocity distribution $f(v, x, t)$ of cars on a homogeneous highway at location x and time t. The terms on the right-hand side of Eq. (1) are given by

$$\left(\frac{\partial f}{\partial t}\right)_{\text{relaxation}} = -(f - f_0)/T, \tag{2}$$

$$\left(\frac{\partial f}{\partial t}\right)_{\text{interaction}} = (1-p)c(\bar{v} - v)f, \tag{3}$$

$$\left(\frac{\partial f}{\partial t}\right)_{\text{adjustment}} = \lambda(1-P)c[\delta(v-\bar{v}) - f]. \tag{4}$$

The various symbols have the following meanings. $f(x, v, t)$ denotes the single-vehicle velocity distribution function, such that $f(x, v, t)dxdv$ is the expected number of vehicles at time t that have a position between x and $x + dx$ and a velocity between v and $v + dv$. $f_0(x, v, t)$ denotes the desired velocity distribution function. λ is the function of density, and $\bar{v}$ denotes the mean velocity of f. The zero-order moment of $f(x, v, t)$, $c(x,t) = \int_0^\infty f(x,v,t)dv$ is the vehicular density. The first-order moment, $\bar{v}(x,t) = \frac{1}{c(x,t)}\int_0^\infty vf(x,v,t)dv$ is the mean velocity. P is the possibility of passing, depending on the density c. T is the relaxation time, which depends on the density c.

3. Lagged cell-transmission model

The Lighthill–Whitham–Richards (LWR) model (Lighthill and Whitham, 1955; Richards, 1956) is a first-order macroscopic traffic flow model. Its basic equation is