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Guaranteed prediction and estimation of the state of a road network

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ABSTRACT

The paper presents an algorithm for the prediction and estimation of the state of a road network comprising freeways and arterials, described by a Cell Transmission Model (CTM). CTM divides the network into a collection of links. Each link is characterized by its fundamental diagram, which relates link speed to link density. The state of the network is the vector of link densities. The state is observed through measurements of speed and flow on some links. Demand is specified by the volume of vehicles entering the network at some links, and by split ratios according to which vehicles are routed through the network. There is model uncertainty: the parameters of the fundamental diagram are uncertain. There is uncertainty in the demand around the nominal forecast. Lastly, the measurements are uncertain. The uncertainty in each model parameter, demand, and measurement is specified by an interval. Given measurements over a time interval $[0,t]$ and a horizon $\tau \geqslant 0$, the algorithm computes a set of states with the guarantee that the actual state at time $(t+\tau)$ will lie in this set, consistent with the given measurements. In standard terminology the algorithm is a state prediction or an estimate accordingly as $\tau > 0$ or $=0$. The flow exiting a link may be controlled by an open- or closed-loop controller such as a signal or ramp meter. An open-loop controller does not change the algorithm, indeed it may make the system more predictable by tightening density bounds downstream of the controller. In the feedback case, the value of the control depends on the estimated state bounds, and the algorithm is extended to compute the range of possible closed-loop control values. The algorithm is used in a proposed design of a decision support system for the I-80 integrated corridor.

1. Introduction

Standard control theory provides a useful framework for formulating and answering questions of real time traffic management. In this framework the evolution of the road network traffic is modeled as a dynamical system,

$$x(t+1) = f(t, x(t), u(t), v(t)), \quad t \geqslant 0, \tag{1.1}$$
$$y(t) = h(t, x(t), w(t)), \tag{1.2}$$

in which $x(t)$ is the traffic state vector at time t. The evolution of $x(t)$ is affected by both controlled inputs (ramp metering, signal settings, changeable message signs) denoted by $u(t)$, and uncontrolled inputs or disturbances (demand, events, weather, incidents) denoted by $v(t)$. The vector $y(t)$ of detector measurements (flow, density, speed, incidents) provides information about the traffic state according to (1.2), in which $w(t)$ is the measurement 'noise'. In the framework, the traffic management strategy is just a feedback function Φ

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$$u(t) = \Phi(t, y[0, t]),$$

which specifies how the control $u(t)$ is selected on the basis of the measurements available up to that time, namely $y[0,t] = \{y(s),\ s \leqslant t\}$.

Suppose we are given the road network model $\{f,h\}$, the feedback function Φ, and a probabilistic characterization of the uncertainties in the forecast demand, disturbances and measurement noise $\{v(t), w(t)t \geqslant 0\}$. Then the prediction of the traffic state at a future time is summarized by the probability distribution of the future state, conditioned on the measurements available at the present time (Kumar and Varaiya, 1986). That is, the prediction of $x(t+\tau)$ is the function

$$\Psi(\xi, t+\tau, t, y[0,t]) = Prob(x(t+\tau) = \xi | y(s), 0 \leqslant s \leqslant t),$$

which is the probability density that $x(t+\tau) = \xi$, conditioned on the measurements $y[0,t]$. The function Ψ summarizes everything one can know about the road network performance under the specified feedback function or management strategy. For example, from Ψ one can calculate the *average* performance of the feedback function in terms (say) of the expected delay as well as the *risk* in terms (say) of its variance. As another example, from Ψ one can calculate the likelihood or probability of the event that congestion will develop during $[t, t+\tau]$. One can then determine whether a proposed management strategy provides adequate average performance and acceptable risk, or whether a proposed strategy improves upon the baseline strategy.

Two difficulties make it virtually impossible to calculate the function Ψ. The first difficulty is computational. To appreciate it, consider a 20 km long two-directional highway, with detectors every 500 m reporting speed and density every 30 s. Then $y(t)$ is a 160-dimensional vector. Suppose the freeway is modeled as a nonlinear discrete-space, discrete-time system, with 500 m links in each direction, with the state as the vector of link densities. Then $x(t)$ is a vector of dimension 80. So Ψ is the probability distribution of the 80-dimensional vector $x(t+\tau)$ which depends on the $80 \times t$-dimensional measurements $y[0,t]$. Computing Ψ is at present impossible. However, with sufficient computational resources, one may be able to calculate more or less satisfactory approximations to Ψ, although to our knowledge no one has attempted to do this calculation. Much more commonly, one resorts to an approximate calculation of the expected value of $x(t)$, conditional on $y[0,t]$, with no attempt to calculate the risk or dispersion of the distribution around this point estimate. As a consequence, one cannot estimate performance measures, such as travel time or delay, which are nonlinear function of the density.

The second difficulty may be more fundamental. The calculation of Ψ assumes that f, g, Φ and the probability distributions of the demand forecast errors, disturbances and measurement errors, are accurately known. This assumption, however, does not hold in practice. The assumed models and probability distributions will have specification errors which must be accounted for in the prediction Ψ. One possible move that overcomes this difficulty is to parameterize the unknown specification errors in a (large) parameter vector Θ, place a prior distribution on Θ, and augment (1.1) with the additional state vector $\Theta(t)$, with

$$\Theta(t+1) = \Theta(t).$$

The prediction function is correspondingly augmented:

$$\Psi(\xi, \theta, t+\tau, t, y[0,t]) = Prob(x(t+\tau) = \xi, \Theta(t+\tau) = \theta \mid y(s), 0 \leqslant s \leqslant t).$$

The computation of this function is thereby much more difficult, and makes this standard control theory formulation more impractical.

Previous work on the traffic state estimation and prediction using macroscopic traffic models consists of variations on the theme of Kalman Filter and Monte Carlo methods. In (Sun et al., 2003), a piecewise linear replacement of the CTM is introduced and the Mixture Kalman Filter (MKF) is used to estimate the discrete and continuous state of the system. In (Tampère and Immers, 2007) the Extended Kalman Filter (EKF) framework for freeway traffic state estimation presented in (Wang and Papageorgiou, 2007) was applied to CTM. The Uncented Kalman Filter (UKF) (Julier et al., 2000), which overcomes some disadvantages of EKF, such as the need for linearization and complicated calculations of Jacobians and Hessians, was compared to EKF in (Hegyi et al., 2006), and it was concluded that their performance was comparable. Particle Filter (PF) approach and its comparison to UKF is described in (Mihaylova et al., 2007). These stochastic filtering techniques rely on assumptions about distributions of the inputs (MKF, EKF), or require large number of simulations to get reasonable result from Monte Carlo methods (UKF, PF): for example, the system with K uncertain inputs and no parametric uncertainty would require a minimum of 2^K simulations to reasonably represent the distribution of the system state.

We propose a different approach for traffic state prediction and estimation, based on set-valued (or bounding) philosophy (Kurzhanski, 1972, 1989; Milanese et al., 1996), that is computationally feasible. We incorporate all of the probabilistic and modeling uncertainty in a (large) parameter vector γ and rewrite the dynamical system as a deterministic system with an unknown uncertainty parameter γ:

$$x(t+1) = f(t, x(t), u(t), \gamma), \quad (1.3)$$
$$y(t) = h(t, x(t), \gamma). \quad (1.4)$$

We assume that we have prior knowledge that the unknown γ belongs to a known set Γ. Instead of the probability distribution Ψ we now seek to find sets $X(t+\tau, t, y[0,t])$ such that, consistent with the measurements $y[0,t]$, we can guarantee that